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ESSAYS ON THE FORMAL THEORY OF PARTIES AND ELECTIONS

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF ECONOMICS
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Luis Fernando Medina S.
August 2000

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Abstract

The essays comprising this dissertation approach different but related topics of the theory of political parties and elections from a formal point of view. The first essay offers a general discussion of two different models of governance that pervade the further analyses. Thus, I compare what I call "constituency-based" regimes with "nation-based" regimes both in terms of their types of legislatures and the main features of the parties that are likely to thrive in them.

The distinction between types of parties, according to the relative weight of their factions, is used in the second chapter to develop a formal theory of endogenous party alignments. On the other hand, the contrast between legislatures with agenda-setting powers and those without it, is used in the third chapter to analyze how each setting leads to different policy outcomes and allocations of political power. Finally, the last chapter allows for abstention and develops a framework to analyze the patterns of participation bias in a politico-economic environment.

Acknowledgements

To say that the genesis of this dissertation was complex and protracted is an understatement. Therefore, above anything else, it is a pleasure for me to write this section of acknowledgements. The many occasions in which the warmth, encouragement and insight of others proved to be just what I needed are among the happiest of all these years and already constitute a wonderful luggage of cherished memories.

Two people that greatly influenced me through their writings (to the extent that, without them, perhaps I would have not pursued advanced studies in economics), were of enormous help throughout the process of writing this dissertation. It is a privilege to add my name to the long list of people whose scholarly production has benefited from Kenneth Arrow. It speaks volumes of his intellectual generosity the fact that over several years he remained as a firm support, even at times at which I could hardly bring myself to believe in this project. John Roemer painstakingly guided me through the details of his then unpublished work on electoral competition and, in so doing, shook my dissertation out of a period of confusion.

Not being myself a political scientist, I wonder how I would have managed to scrape the surface of such a profuse discipline were it not for the unfaltering guidance and insight of John Ferejohn and Morris Fiorina. Roger Noll pointed out so many ways in which I could strengthen my analyses that, as much as I have tried to absorb some of them, enough remain to keep me busy for a long while. With sharp suggestions and contagious enthusiasm, Masahiko Aoki has enriched my understanding of my own ideas in ways I always thought were only possible after a much more protracted interaction.

Ignacio Ortuño-Ortín revived my long-standing interest in political economy, at the beginning of my dissertation years while, toward the end of them, Douglas Grob became the first living evidence that my ideas could interest political scientists and prompt them to share their knowledge with me. The list of people who, at one time or another, helped by wading through my drafts, offering suggestions and puncturing mistakes is too long to warrant any attempt at recreating it here. But, at the risk of egregious overlooks, I will mention David Baron, Douglas Bernheim, Johnathan Bendor, Manuel Hidalgo, Keith Krehbiel, Robert MacMillan, Nolan McCarty, Kenneth Shepsle, Matthew Shum, Valter Sorana and Barry Weingast.

Before coming to Stanford, I was fortunate to know the vibrant academic community of Colombia. Here I wish to mention especially Manuel Ramírez who did more than anybody else at the time to introduce me to the real richness of rational choice theory. The Banco de la República de Colombia and Colciencias provided world-class financial aid. More than a paycheck, every month, rain or shine, I received from my country a touching manifestation of faith in my capabilities.

I can hardly think of an environment more stimulating than that of Stanford University. However, sometimes its superb standards of scholarly achievement can become intimidating and even paralyzing. At those moments, no amount of intellectual exchange is enough. Whoever said that there are no friends like those of childhood never met someone like Marcelo Bucheli. Throughout these years, to every problem that came upon me, there was a positive side: the chance to see him come through, adding a handful of laughter in the process. As much as I owe Stanford a huge intellectual debt, one of its greatest legacies for me will always be the friendship of Klaus Desmet, Fabiano Schivardi and Matthew Shum who never failed to provide whatever amount of hand-holding and advice I asked from them, always with sincerity and good cheer.

During the years of writing my dissertation, I was parted by one life, and joined by another. Through them I want to convey a tiny fraction of what my family has meant for all this long process. In the midst of a particularly difficult time for my thesis, my father passed away unexpectedly. He did not live to see the final text we so much dreamed with.

Even the most grandiose one, would have been little retribution for all his effort and, at the same time, even the most humble one would have gratified his love for me. Fortunately, while the same is true about my mother and my sister, they are with me now so that we can share the culmination of this stage.

My daughter, Alejandra, was born during my Stanford years. By bringing beauty and joy to all those around her, in just a couple of years she has made the world a better place in ways the present pages can only dream about.

In retrospect, it is no miracle that this thesis is now finished. After all, it is the result of the concerted effort of many people, deeply committed to this cause. People for which no sacrifice was too big and no detail too small. They served as academic interlocutors, constructive critics, financial assistants, childcare providers, bread-earners and many, many more roles. What is a miracle is that all these people were one and the same loving person: Alejandra's mother, my wife, Claudia.

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Chapter 1

Introduction: Political Parties and Governance Regimes

Although conceived independently, the essays comprising this dissertation share a common methodological background and a common substantive core. While it may be clear, even for casual readers, that the method adopted here is that of rational choice theory, the connections between the subject matters of the essays may not be transparent, not the least because of their genesis. I will spell out in this introductory section what I believe is a unifying framework for the dissertation, but, in so doing, I must confess that such a framework was not entirely perspicuous to me while writing it. In fact, it took me some rereading of the final project (and the insights of a splendid dissertation committee) to visualize the different pieces as parts of a larger project in comparative political economy.

1.1 Programmatic Overview

Universal franchise is one of the deepest social innovations in history. In the nations in which it has been introduced in a durable manner, it has changed the everyday life of its inhabitants to an extent to which few technical inventions can aspire. In fact, it redefined down to the roots the way in which individuals (now called citizens) relate to the State. However, the patterns of the transformations it has led to in different societies are quite bewildering. "Political democracy" is not a uniform category any more than "market economy" is.

This is no accident. The causes for heterogeneity among market economies and democracies are somewhat similar. In even the tiniest nations, the tasks of exchanging goods and making decisions in the public arena cannot be handled without recurring to some complementary institutional arrangements. Just as markets coexist with "islands of central planning", called firms (the point cleverly made by Coase), one-citizen-one-vote democracies require other decision-making mechanisms, like delegation and centralized authority.

Tolstoy once quipped that happy families are all alike whereas each unhappy family is miserable in its own way. This seems to be the point also of the "new institutionalism". Societies populated by perfect, omniscient agents, would be all alike, that is, their economy would be a pure market economy, permanently operating at the general competitive equilibrium and their polity would function by a mixture of deliberation and direct voting among all the citizens, without any need for other institutional accoutrements.

On the other hand, real-life societies display a vast diversity on both counts. Given that pure, unfettered deliberation among all the citizens is not feasible, democracies have developed different types of intermediaries between the individuals and the State. In the essays that follow, I single out two of them: the political parties and the legislatures. I believe that the patterns of their (sometimes uneasy) coexistence holds the key to understanding many of the most important features of different democracies.

While legislatures historically predate universal franchise, the modern mass parties are a product of it. True, there was no shortage of partisan activity in parliaments before the extension of suffrage (as is attested by the Whig and Tory split in XVIIIth Century England) but by no standard do these factions resemble the modern parties that operate in any democracy.

Universal suffrage changed parties, and in the process, their relationships with legislatures. Without any claim to completeness, I want to highlight several mechanisms through which this occured. Mass elections created both a challenge and an opportunity for political entrepreneurs. The challenge was to reach out to an enlarged electorate, coming from all walks of life, some of it living in remote regions, some of it completely oblivious to things political, some of it illiterate and so on. The opportunity was that, whoever was crowned by success in this enterprise could, in principle, tap into an entirely new source of political legitimacy.

Old style parliamentary parties were unable to meet the challenge or to benefit from the opportunity. Without a solid organizational basis, this parties were not up to the task of garnering votes in a mass election. On the other hand, they had little, if any, existence independent of the legislature and thus, could not lay claim to any legitimacy different than that of the legislators themselves. This explains why the difference between modern mass parties and old parliamentary factions is of nature, not just of degree.

To some extent, this transformation of parties from legislative clubs to nation-wide, grassroots organizations, amounts to implanting a foreign body in the governance arrangements prevalent up to that point. In particular, it opens alternative mechanisms of coalition-building. Whereas legislators had been historically accountable to their constituencies, the party organizations transcend the geographical barriers of such constituencies. They become the vehicles through which citizens of different districts can coalesce with the aim of controlling the levers of the State. If I had to choose one single lesson that I learned in preparing these essays and that I would want to communicate is that it is hard to overrate the importance of this tension between constituency-based and nation-based models of governance. The particular way in which this tension is solved within each country dictates, to a large extent, the main characteristics of its institutional makeup.

In some cases, the parties have been merely superimposed over the old governance structure, without altering it fundamentally. Instead of trying to recreate the landscape of political coalitions, these parties have tried to use the existing, constituency-based structures as their springboards to national viability. In few cases is this more clear than in the "clientele" parties that exist, for example, in some Latin American countries. At the extreme, these parties are an enlarged version of the parliamentary clubs, resting over a large organizational base that serves as the transmission belt between the elected officials and their constituencies. Little, if any, attempts are made at bringing additional elements of coherence to the club.

In other cases, the parties have actually brought about a recomposition of the coalitional structure of the polity. At the extreme, we could place the typical "strong", class-based parties of some countries in Continental Europe. When they are at their most successful, these parties manage to replace the constituencies as sources of legitimacy, to such an extent that the legislature becomes little else than a rubber stamp for the electoral mandate.

This is not the right place (and I am not the right person) to speculate about the causes of these patterns. Instead, I want to discuss their implications for the organizational structure of parties, the stability of the party systems and the allocation of power across society.

Like any other organization, political parties need to display a good fit with their environment if they are to survive. Therefore, depending on the circumstances under which they operate, parties will differ in the amount of resources they devote to the several tasks they are supposed to perform. As the circumstances change, some partisan skills become more valuable than others and so, the organization, if it is to survive, will need to attract more of the most valuable resources and less of the others. A party that is entirely out of power, operating under political persecution, has little use for a defty parliamentary deal-maker. Instead, it may badly need brave agitators and even some expertise in undercover operations.

By the same token, the tasks a party needs to perform differ substantially if it is to behave as an extension of a parliamentary club or if it wants to replace it with new, inter-constituency, coalitions. Under the first scenario, the party needs to enhance the electoral prospects of its legislators since they are the very backbone of its organization. Under the second scenario, it needs to coordinate the efforts of activists and voters that, while sharing common ideological persuasions, are dispersed across the electoral geography of the country. I would conjecture that this second type of party will depend more heavily than the first one on the ability to articulate well-defined programmatic platforms that serve as "focal points" for its rank and file. In sum, whereas the first type of party will behave more as a vote-maximizer, the second will resemble more the ideological party postulated in much of the comparative literature on parties.

The first essay on "A Theory of Endogenous Party Alignments" draws on these organizational differences across parties. The distinction between "militants" and "opportunists" suggested by Przeworski and formalized by Roemer is used here to assess the way in which parties balance their needs for ideological coherence and electoral viability. Although the terminology adopted may seem somehow value-laden, it should be clear that both types of party activists perform tasks important for the survival of the organization. A party that sacrifices every electoral consideration to preserve some ideological purity is likely to disappear. On the other hand, a purely opportunistic party will fail to generate credibility among voters. However, as argued above, the actual weight attached to each of these tasks will depend on the environment in which the parties operate. According to the conjecture formulated in the last paragraph, constituency-based systems of governance generate incentives for the opportunist factions of the parties to thrive at the expense of the militant ones. On the other hand, the services provided by militants have a higher relative value for the parties in nation-based systems of governance.

This dichotomy between models of governance, which is developed in an explicit manner in the second essay, leaves unspecified some aspects which are the main subject of the essay on party alignments. In order to create cross-district coalitions, parties need to mobilize geographically dispersed voters around certain common grievances. Typically, societies are subject to various political cleavages but only a few of those can become activated through the parties. Since numbers are of the essence in a democracy, coalitions must be built and this requires that citizens forsake certain claims in favor of others. The process through which some dimensions of social conflict gain prominence in the political arena at the expense of others is what generates party alignments.

One major question raised by the comparative analysis of party systems is that of the stability of such party alignments. Countries differ as to the durability of the factional structure underlying their parties. In some countries, as in the US previous to the current era of divided government, the parties tend to undergo periodic overhauls, manifested through massive reshuffling of the coalitions and even voters' revolts against the established partisan structures. On the other hand, countries like the major democracies in Western Europe (at least, once again, until the 80s) seem to be rather immune to such convulsions.

The main result of this essay is that the susceptibility of a party system to realignments induced by shifts in the voters' preferences depends on the organizational structure of the parties themselves. The stronger the militant factions of the parties, the more impervious is the alignment to changes in the relative salience of the different policy dimensions.

This result may become more suggestive when considered together with the main thrust of the second essay "Legislatures vs. Political Parties". There I analyze how the behavior of voters and parties lead to different outcomes as we move from a system of ample policy-making powers for the legislature to one in which this body's main role is to ratify the results of the general election. In the light of the preceding paragraphs it should come as little surprise that in the first case parties play a merely subsidiary role in shaping policy outcomes, completely unlike what happens in the second case where they become the most influential force in the polity. However, this essay tries to go beyond this main idea by analyzing how this pattern is reinforced by the strategic decisions of the voters and by providing a characterization of the type of policy outcomes and legislative behavior that will obtain in each case.

It is important to notice that, within this framework, there are several interactions between the political geography of the electorate, the organizational structure of the parties and the constitutional arrangements under which the legislature operates. In fact, it can be argued that the three variables reinforce each other.

At the end of the second essay I show that the impact of the restraints imposed over the agenda-setting powers of the legislature (i.e. the "closed-rule", in the terminology employed there) is larger the more heterogeneous the population within the districts. I believe these are the same conditions that give incentives for the parties to try to create cross-district coalitions. If districts are purely homogeneous, the parties cannot offer to the voters a model of representation that improves upon the one provided by the constituency-based legislatures. Now, in that case, as said before, the presence of the militant factions within the parties will be relatively stronger. On the other hand, these are the same parties that are more likely to gag the legislatures by depriving them of agenda-setting powers. This, in turn, will imply that, when in power, they are in a better position to deliver the policies their rank and file voted for, thus reinforcing the loyalties that keep the organization together. In other words, it appears as if there are several complementarities between heterogeneous (usually, large) districts, ideological parties and weak legislatures. By the same token, (small) homogeneous districts, electoral parties and strong legislatures seem also to go together and reinforce each other through analogous complementarities. I claim that, if this set of conjectures is true, then the distinction proposed here between constituency-based and nation-based governance can be used to provide an operational typology of political regimes, one that can lead to fruitful comparative analyses.

1.2 Methodological Issues

Thus far, in trying to insert these essays within a broader research program, I have tried to be as exoteric as possible, avoiding technical references to the rational choice paradigm. This is deliberate. I think that the type of comparative program I have sketched here ought to incorporate the wealth of knowledge accumulated by other traditions. (In fact, I think of the preceding observations as a call to integrate the older traditions of political sociology and organizational theories of parties with modern comparative institutional analysis.) True, I also think that the rational choice approach is the best candidate to provide a unifying language to this knowledge and, accordingly, this is the method I have used here. However, in their current state, the models presented here are not adequate for such an ambitious enterprise. They are simply a first foray in the field. So, this section will discuss some of the technical issues that must be dealt with in order to strengthen the formal framework developed here.

Given the dearth of formal analyses of the party alignment problem, it is not surprising that the first essay ends up leaving several modelling questions without answer. I believe that the solution concept used there (viz. stable, one-sided matching) provides a good first approximation to the problem of party alignments. However, apart from its analytical difficulties, it remains silent about other patterns of alignment that can emerge in real polities. As mentioned in that essay, a property of one-sided matching problems is that they may lack stable solutions when all the agents rank the same potential match as the worst. But this is true only if we restrict ourselves to solutions that generate pairs of matched agents. The situation may be different if we allow for richer structures of matching (say, three agents). In fact, it may be argued that in many polities, large, multi-faction coalitions come into existence precisely because there is one faction that is rejected by the rest. Far from being non-equilibrium situations, these type of alignments tend to be quite stable. (One is tempted to think in these terms of the long-lasting prominence of the Italian Christian Democrats as other parties tried to distance themselves from the Communists.)

Another limitation of that model is its restriction to two-dimensional spaces. Of course, this can be defended on grounds of simplicity, legitimate if we are to begin the study of a problem as is the case here. But for future research, this opens an interesting avenue of inquiry. A tacit assumption made here is that the militant factions are willing to cooperate with others along one dimension of the policy space since this is what allows them to become viable parties. However, the definition of viability is itself institutionally dictated. In this essay, since the underlying decision-making mechanism is pure majority rule, the hurdle of viability is high so that isolated factions are in risk of being wiped out of the political landscape. On the other hand, if we allow for other institutional arrangements (like PR systems) that same hurdle is lowered. Then, it is quite possible that the incentives for the factions to compromise some of their goals in order to cement a coalition with others, are weakened. (Notice that this argument is reminiscent of the "minimal winning coalition" principle.) In other words, in the model presented here, whereas the policy space is two-dimensional, partisan competition is one-dimensional but, this is, to a large extent a result induced by the majoritarian nature of the institutional framework. What is the "right" dimensionality of the partisan competition is something that, in all likelihood, depends precisely on the type of decision-making institutions prevalent in a polity. As Gary Cox [1] observes, nobody thinks that the US would remain a two-party system were it to adopt the Israeli electoral code.

Most of the modelling choices in this essay are related to the difficulties associated with multi-dimensional policy spaces. So, it is important to keep in mind other alternative solution concepts for the same case. The reader familiar with the literature on European parliaments will recognize that the problem formulated here is similar to that of cabinet stability studied by Shepsle and Laver [3]. On the other hand, the equilibrium concept utilized in that model (viz. the "win-set"), while appropiate to study the formation of cabinets whose survival depend on their ability to fend off attacks from would-be majorities, it may not fully capture what occurs at the intra-party level. There is a meaningful way of saying that factions vote in the legislature whereas such concept becomes blurry in elections: political factions do not vote in elections (or, more precisely, they do but their numbers are not decisive). Instead, they try to garner the necessary votes from among the citizenry. Therefore, in the process of "making and breaking parties" (to paraphrase the title of the book by Shepsle and Laver), the factions need to consider how their decisions will affect their further ability to bring voters to the polls. In other words, the cabinet-formation problem can be solved entirely without assuming the presence of opportunist factions, something that may seem quite implausible in electoral models. This implies that the hurdles for an equilibrium of the intra-party model are higher than for the cabinet model.

Of course, the strengths of the one-sided matching model for problems of intra-party coalition formation become weaknesses when we try to apply it to problems of cabinet formation. The tightness of the constraints imposed on an equilibrium become implausible and, hence, an analytical liability. Thus, I think that, instead of trying to take sides for one concept or another, a wiser strategy would be to see how to make them complementary.

The second essay, on "Legislatures vs. Political Parties" also involves some methodological issues. To my mind, the most important is the concept of strategic voting used there. By strategic voting in this context I mean the voting decisions that are taken by considering not just the ideological views of candidates but their impact, if elected, on the policy outcomes, given the information the voter has about the decision-making rule of the legislature. I believe that this type of strategic voting is the natural microfoundation for the type of analysis spelled out in the preceding section.

On the other hand, the model developed there still needs some further elaborations if it is to provide deeper insight on the issue of to what extent parties attain preminence over the legislature, if at all. In this respect, a direction in which this model should be extended is to enrich the description of the candidate-selection process at the district level. In real life this is a crucial moment that decides the extent to which parties are able to reign in over their elected officials. The simplifying assumption of candidates that do not have to go through a nomination process, while helpful in bringing about crisp results, overlooks this aspect.

The third essay on "Some Properties of the Probabilistic Voting Model" is somewhat different in nature to the other two and hence, it needs some special comments. Probabilistic voting has emerged recently as an influential paradigm in rational choice models of elections. As such, it holds promise of providing a sound framework for the study of electoral abstention, one of the most important phenomena in any democracy. This third essay has a dual purpose. At a methodological level, it tries to probe the extent to which the extant literature has fulfilled such promise. In that sense, my answer is quite skeptic. I believe that the issue of which should be the microfoundations of probabilistic voting still plagues even some of the best pieces in the field. Such is the spirit of my comment on the very influential work by Enelow and Hinich [2]. A conclusion to be drawn from this analysis is that deterministic models are not robust. "Small" levels of uncertainty lead to very different results. In particular, the median-voter theorem does not hold in a probabilistic setting or, to be more precise, only holds under very special assumptions. Probabilistic voting is not simply an extension of deterministic voting, but a competing theory.

At a substantive level, the goal of that essay is to ponder how can the probabilistic models be put to use in the analysis of the socioeconomic biases of turnout, a classical theme of the literature. In this respect, I think that probabilistic models can provide useful, operational hypothesis but I think that further research is needed before we have a satisfactory rational choice model of turnout. In particular, a possibility worth exploring is that of strategic voting. In a multi-district context, candidates for a legislature become complements for the voters: there is little gain for, say, extremist voters in a district to mobilize and to elect "their" candidate if voters of similar views in other districts are not doing the same; their efforts would just amount to send a lonely voice to the legislature. If we consider this, it is possible that a more explicit account of the multi-district setting will provide a sounder base for the analysis of voters' mobilization. In the light of this, the third essay may be considered (in fact, was meant to be) an attempt of getting the most out of the single-district case, before daring into uncharted territory.

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Chapter 2

A Theory of Endogenous Party Alignments

2.1 Introduction

One of the most important functions performed by political parties is that of *aggregating* the multiple lines of conflict within a heterogeneous citizenry. Without this reduction in the dimensionality of conflict, representative democracy as we know it would be hardly feasible, if at all. No complex society can afford to grant equal stretches of deliberation to each and every possible conflict that arises at any given time. Some issues must yield to others in order to attain, not only meaningful debates, but also (more relevant to the present paper) effective coalition-building.

In political science, the question of how the different sources of conflict become politicized and addressed in the electoral competition has become part and parcel of the study of *party alignments*. Clearly, party alignments are multi-faceted phenomena and many different approaches can be (and have been) taken to analyze and understand them. Somehow surprisingly, one approach that has not been fully utilized in this connection is the one of rational choice theory, akin to the methods of economic analysis. The present paper is an attempt to study the problem of party realignment from such a perspective.

Briefly, the argument runs as follows: the aggregation of different conflicts that leads to partisan alignments in a democratic polity imposes over the parties and their supporters a series of trade-offs whose magnitude will depend *both* on the preferences of the electorate and on the institutional underpinnings of the political parties. Whereas voters' preferences constitute, as it were, the *demand* side of party alignments, the internal structure of the parties places constraints on the *supply* side. The analysis of this interplay is the main goal of this paper.

The structure of the paper is the following: Section2.2 provides some background about the topic of party alignment, as it has been addressed in the political science literature, discussing its major findings, conundrums and weaknesses. Section2.3 addresses the limitations of spatial models that prevent an analysis of party alignments from a game-theoretic perspective. Likewise, it argues for the use of John Roemer's recent proposal (Roemer [6],[24]) of *Party Unanimity Nash Equilibrium* (henceforth *PUNE*) as a way to alleviate most of these limitations. Section2.4 presents the basic model and its results. Finally, Section4.5 offers some conclusions. Lengthy proofs are relegated to the Appendix.

2.2 Party Alignments in Political Science

The analysis of party alignments has not been an exception to the general trend within political science of, so to speak, a North- Atlantic split. That is, the outlook and sometimes even the methods used, differ substantially from the US studies to those about Western Europe. A rather early stage of the European understanding of party alignments matured with the collective volume edited by Seymour Lipset and Stein Rokkan in 1967 (Lipset and Rokkan [7]) and the influential introductory essay coauthored by these. Two major points from this essay are of concern here. First, LR's analysis was essentially socio-historical: their main goal was to substantiate the claim that the party alignments in Western Europe, that is, the major lines of political competition, were dictated by the underlying structure of social conflicts and, even more substantively, by the *timing* of these. In their scheme, there are three major conflicts that all the polities in their analysis have gone through in their process of nation-building: the State-church conflict, the center-periphery struggle and the economic dimension be it either as a confrontation between producers and consumers and/or between capital and labor. In this connection, their essay argued that discernible cross-national differences in the party alignments in Europe could be traced back to the actual sequence in which these different conflicts were addressed.

After a detailed comparison of the major nations in Europe, they arrived to a second, controversial and bold conclusion: the party alignments in these countries had *frozen* around the 20's, that is, by the time the process of universal enfranchisement was complete. In other words, the political families of parties in the late 60's Europe were the same of those active in the 20's. Doubtlessly, it is a daring hypothesis. These were not just *any* four decades, these were, arguably, the most tumultuous period in centuries for many of these countries. That the party alignments of the 20's had survived mostly unmodified the impact of the onslaught of Bolshevism and Fascism, a major economic depression, the largest war ever recorded in history and the subsequent reconstruction is surely cause for bafflement.

Ironically, shortly after the publication of this book, events in Europe seemed to suggest that the party systems were actually beginning to thaw. The early 70's witnessed in some countries (notably, Danemark, Holland and Britain) signs of what Alan Ware (Ware [14]) has called voters' revolt against their well-established parties. The Progress Party in Danemark shook the electoral climate in 1971 while the gains of the British Liberals in those same years suggested that something as momentous as the downfall of the same Liberal party following the famous 1918 Election was already brewing. Now we know that this massive shifts of those party systems did not materialize. By 1985, in a comprehensive study of Social-Democratic parties in Western Europe, Herbert Kitschelt concluded that a shift in the axis of politics was under way. From a period of "economic" politics, of electoral competition around issues of income redistribution, Kitschelt claimed that Europe was moving toward a period of "social" politics, marked by conflicts over the role of the State in regulating aspects of the citizens' private lifes (notably, marriage and reproduction). Paralleling this shift he argued for a shift in the underlying Social-Democratic coalition in such a way as to include middle-class strata, historically left out. There is a case to be made in favor of this interpretation, specially with the performance of the left-of-center parties in Europe in the second half of the 90's.

Be it as it may, another wave of revolts was identified in the early 90's this time with the additional ingredient of the splits around Euro-integration and the end of the Cold War. Once again, those bracing themselves for a political earthquake have been disappointed but, it is only fair to say that the jury is still out as for the long-run consequences. Altogether, although the "freezing" hypothesis is no longer as powerful as it might have been when first suggested, it still retains something of its sharpness.

The dynamics in the US has gone, pretty much the other way around. Massive realignments used to be the staple of American politics at least since the collapse of the Whig party at the outset of the Civil War, torn from within over the issue of slavery. By the 1970's American political scientists had already produced a substantial literature on *critical elections*, understood as elections where an abrupt break in the prevailing party system occurred. The elections of 1860, 1896 and 1932 were, head and shoulders above the rest, regarded as such realigning elections: previous to all of them, both major parties had undergone profound transformations in their factional makeup, recruiting new sectors of the electorate, alienating old ones, changing their geographical turfs and, altogether, transforming their outlook and what they stood for vis-a-vis their voters. Not for nothing, Walter Burnham (Burnham [3]) considered these realigning elections, as the American surrogate for revolution. There was still some further room for discussion about other elections (1916 and 1952 come to mind) and their potential realigning effect[1]. Inconclusive as the arguments could be, there seemed to be one consensus: the American party system appeared to be much more in flux than its European counterparts.

So changing was the landscape of partisan politics in the US that some political scientists, as they gained confidence in their theories, began to extrapolate the trend of a critical election every 30-something years (in a state of affairs that may remember to the economist the business cycle studies of the pre-WWII era). Hence, the *prediction* of a critical election

[1]The impressive array of data mustered in Sundquist [13], still makes it a landmark reference on this history, in spite of recent challenges like that of Poole and Rosenthal [8]

somewhere in the 1960's. For all the turmoil of those years, the 60's failed to produce elections that could anyhow meet the standards of "criticality" set by the top-ranking examples. Something similar could be said of elections after this period, which is not to say that the political seismographer has been steady; 1968, 1980 (being 1994 still too recent) marked some important landmarks. Moreover, as Poole and Rosenthal suggest, there is every reason to believe that a silent realignment process is under way as the last vestiges of the old "Democratic South" fall in Republican hands, thus ending what they have astutely defined as the American three-party system (Poole and Rosenthal [8]).

All things considered, it is tempting to say that politics in the advanced democracies is slowly converging to a state of moderate instability. While Western Europe seems to be "waking up" from the slumber postulated by Lipset and Rokkan, the US is "lulling down" in a smoother pattern where critical elections are being substituted for by divided government.

Is there anything to these two different paths? Where should one look for explanations? Although I cannot claim having an answer for such a daunting question, in the present paper I will argue that one aspect should not be overlooked: in thinking about political realignment (or the lack thereof) one must not settle down for a purely sociological account of the underlying lines of conflict, one must also pay attention to the *institutional structure* of the political parties themselves. True, social tensions go a long way in explaining the amount and type of parties in a polity as Lipset and Rokkan claimed 30 years ago and more recent studies have confirmed (see for example, the results reported by Cox [8]). But the dynamics of party systems owes much to the responses of parties to the requirements placed by the voters as they try to aggregate their differences into manageable clusters of political dissent.

2.3 Party Systems in Spatial Models

Although far from unanimous, there has been a consensus in the tradition of spatial models of voting around two major pillars: the modeling of parties as unitary actors (mostly electorally-oriented) and the restriction to one-dimensional policy spaces. For all the merits of these two assumptions, they quickly become an impediment if we try to use spatial theory as a guide to understand the problem of party alignments.

As claimed in the introduction, one of the main roles of party alignments is to provide the political agents with opportunities for coalition-building. In that sense, when we observe a particular party alignment, we are observing the outcome of decisions taken by different actors as to which partners to choose and, hence, which parties to join. A model of unitary parties will not provide insights as to how the party itself came into being, precisely the problem that needs to be addressed in this connection.

On the other hand, since party alignments aggregate lines of conflict in a society, we need to keep in mind that a given party system is a *response* to a situation in which the electorate is split over several cross-cutting issues. Assuming a one-dimensional policy space is therefore not a viable way of analyzing the genesis of party systems.

In that sense, this paper represents a double departure from the orthodoxy. However, a similar departure has already been attempted by John Roemer in his most recent papers so that here I will heavily draw on his contribution. There, Roemer proposes a solution to a long-lasting problem in the spatial theory of electoral competition: in models with multi-dimensional spaces, Nash equilibria for a game between electoral parties generically fail to exist. It is fair to say that this non-existence of a satisfactory solution concept has been the major obstacle to dropping the unidimensionality assumption in spite of its recognized inadequacies.

Technically speaking, the problem of non-existence of equilibrium in these electoral games is related to the absence of Condorcet winners in multiple dimensions. In essence, Roemer's proposal amounts to consider a partial preference ordering that strictly contains the usual one so that the new best-response sets are larger, thus ensuring existence. How then, to specify such a partial ordering? From an organizational point of view, it makes sense to think of parties as formed by members with different, conflicting goals. A case can be made that the two major goals that a party must take into account are its electoral success and its ideological purity. So, one can think of parties as composed of factions that want to further each of these goals: an *opportunist* faction concerned only with the probability of victory and a *militant* faction whose objective is to get the party to remain as close as possible to some particular ideological stance.

In principle, one may ask how should the party play this balancing act. I will have more to say in this connection, but for the time being, let's focus on the following point. Regardless of the particular intra-party decision rule adopted, one should expect that the party's platform is such that it is not possible to attain with an alternative platform a higher probability of victory *and at the same time* a more "ideological" stance. Clearly, were that to be the case, all the members of the party, would unanimously favor that alternative. This gives rise to a partial ordering like the one we mentioned: there will be a set of platforms that strictly dominates the rest of the options but among which there is no consensus within the party members as to which one is better. Some platforms will be preferred by the militants, others by the opportunists. Hence, the requirement of the concept of PUNE is that the parties choose platforms that belong to this choice set induced by their incomplete preferences. The probability of victory associated with a given platform will depend on the platform chosen by the other party. Therefore, the choice set itself will be a function of the rival's strategy. It is in that sense that we can talk of a *Nash* equilibrium in this context: each party's platform is optimal with respect to its incomplete preferences, taking as given the other party's strategy.

On the other hand, notice that, in general, since any element of this choice set meets the requirements of an equilibrium, there will be a continuum of such solutions. In other words, the graphs of the best-response correspondences strictly contain the graphs that would exist under complete preferences over the same domain of probability of victory *cum* ideological purity. If we want to sharpen the prediction of this solution concept, we need to specify an intra-party decision rule that tells us how does it adjudicate the conflicts between militants and opportunists. But this step requires special care. It is also a fact that under complete preferences (the type of preferences we would obtain were we to specify some marginal rate of substitution between probability of victory and ideological purity), Nash equilibria in pure strategies generically fail to exist. In other words, the moment we try to specify further the internal decision rule of the parties, the non-existence issue will crop up again[2]. How to obtain a refinement of the set of PUNEa is still an open question. In what follows I will derive results that hold regardless of the particular properties a possible refinement. There is an obvious price to be paid for this in that these results will not be as sharp as one could expect were there to be an airtight method of narrowing down the predictions of PUNE.

2.4 The Model

For the sake of simplicity, I will consider the case of two parties and two dimensions. As pointed above, parties are considered as heterogeneous: they are coalitions of militants and opportunists. On the other hand, the essence of the problem of party alignments requires us to allow for the militants to have conflicting preferences. As a matter of fact, as will be discussed below, a party alignment is defined by the way the militant factions coalesce, that is, by the dimensions along which these factions choose partners. Just as the problem of party alignments is simply ill-defined in a setting of one dimension, it also makes little sense if the parties' militants form an entirely homogeneous bloc. Each party will be formed by a coalition of two factions of militants and a faction of opportunists. The policy space will be bounded, represented by the set $[0,1] \times [0,1] \subset \Re^2$. The four types of militants can then be described by their ideal points $((0,0);(0,1);(1,0);(1,1))$. Although in principle any pair of militant factions can form a party, I will further restrict the model to consider only coalitions of *adjacent* factions. This is equivalent to requiring that the members of a party must share their views on at least one policy dimension[3]. In order for the equilibria

[2] John Roemer (personal communication) has shown this thoroughly for the case in which we assume that the parties adopt a Nash bargaining solution conditional on the other party's strategy.

[3] I do not think that the losses of generality implied by these choices are significant enough as to invalidate them. It is true that one should expect substantial differences between the results of a two-party model and a multi-party one. But in the case of three or more parties is extremely difficult to characterize so I believe that we should learn whatever there is to be learned from the simple case before daring into the more complex ones. Roemer [24] has simulated a 3-party model for Germany and Sweden in the interwar years. As the reader may appreciate there, this is not a case for the "computationally faint-hearted". Restricting the parties to two types of militants is somehow *ad hoc*, but greatly enhances the tractability of the model. As for the choice of dimensions, first of all, the important discontinuity in terms of location models is to go

of the game to be computed we need to stipulate the pay-off functions of both militants and opportunists. For the case of the militants, I will assume that they have well-behaved Euclidean preferences over the policy space. In other words, for a militant with ideal point (x_i, x_j), her preferences are:

$$U_{ij}(x, y) = -\frac{1}{2}[(x - x_i)^2 + (y - y_i)^2]$$

As for the pay-off function of the opportunists, since their objective is to maximize the electoral prospects of the party, we still need to wait until a full specification of the electoral model is developed (in the next subsection).

An *alignment* will be defined by the factional make up of the parties. Formally, this is represented as the partition of the set of militants generated by a particular party structure. Thus, we will be dealing with two possible cases of alignment: one alignment along the X-dimension (that is, where factions $(0,0)$ and $(0,1)$ form a party and the other two form the other party) and an alignment along the the Y-dimension (where the parties are formed by the partition $\{(0,0);(1,0)\};\{(0,1);(1,1)\}$).

There are several implicit assumptions in the usage of PUNE that are worth spelling out. First and foremost, the concept of PUNE treats both militants and opportunists as indispensable for the existence of a party. I believe this is entirely justified. Parties with no concern whatsoever for their electoral viability can hardly be called parties. They resemble more a sect and, so to speak, harvest what they plow: political insignificance. On the other hand, it is also plausible to think of militants as providing essential services to the organization of a party. Recruiting, campaigning, sitting in long political meetings late at night, etc. are all activities that require some special motivation. In addition, a purely opportunistic party would run into serious troubles of credibility which would precisely undermine its long-run prospects.

from 1 to 2 dimensions; the n-dimensional case seems not to present major additional difficulties. On the other hand, policy spaces tend to be of very low dimensionality to begin with. As to the boundedness of the space, it will be seen that no fundamental mathematical aspects hinge on it.

Furthermore, here I will assume that the alignment is determined by the decisions of the *militants* to join or defect certain parties. To that extent the opportunists will play a rather passive role. Although analytical convenience is the main reason for this choice, there is also a substantive point involved, one that deserves special mention. Political realignments, as said before, are changes in the underlying structure of the parties. As such, they should be distinguished from mere changes in the composition of the government. Electoral landslides are neither necessary, nor sufficient to produce a political realignment. This is a well-known fact among, for example, the US political scientists: the 1964 election returned President Johnson to the White House after a victory of proportions not seen since 1932 (and hardly matched afterwards). However, for all we know, no meaningful *long-run* change was effected over the party loyalties of the voters, over the main issues of contention in American politics, over the coalitional structure of the parties, in one word, no realignment. True, within the framework adopted here, the decisions of the opportunists could bring about serious changes in the electoral fortunes of the parties (for example, if all the opportunists of a party abandoned it, reducing it to electoral insignificance), moreover, in a dynamic setting, there is no doubt that this history of electoral performances of the parties will ultimately play a role over the incentives to align and realign. But, for these changes to take place, it is needed a transformation in the structure of the party as determined by its *militants*. So, since in this paper I will just consider a static framework of realignment (obviously a limitation of the model, imposed by simplicity), the leading role in the process is that played by the militants.

As stated in the introduction, the main thesis of this paper is that political (re)alignments result from the interplay between the voters' preferences, the electoral opportunities they offer to the parties and the ability of the parties to react to those opportunities. When deciding along which party to align, the political entrepreneurs are facing several trade-offs in a multi-dimensional setting: they must decide which of their views will be fostered in the electoral arena and which must remain as mere *intra-party* sources of disagreement. To capture this, I model party alignments here as two-stage games: in the first stage, the militant factions must decide *which* parties to form, that is, over what dimensions will they agree with their competitors. In the second stage, the parties thus created compete in elections. Therefore, when forming parties, the factions will need to consider how do they rank the different outcomes of elections under alternative alignments. This means that, as usual, the game will be solved by using the criterion of backward-induction. As is customary, I will analyze first the second stage, so as to use its results as inputs for the factions' calculations in the first one.

2.4.1 The Electoral Stage

There are several modeling decisions that have to be made when analyzing elections in multi-dimensional spaces. The first one refers to the *maximand* of the parties. The two most common choices are either the probability of victory or the expected vote. However, a well-known fact of electoral models is that in a two-party game they lead to the same results[4]. Here I will adopt the framework of expected vote for a reason that will become apparent later: the way I construct the expected vote function is crisp enough to obtain results and, although it could be used to construct a function of probability of victory with no major difficulty, this last step would constitute an unnecessary detour[5].

The second important modeling choice refers to the composition and behavior of the electorate. Here I will assume that there is a continuum of voters whose ideal points are distributed on the policy space following a bivariate uniform distribution[6]. With respect to the voters' behavior, we need to face a dilemma: the standard practice in voting models is to consider the voter as choosing between alternatives ranked by its *Euclidean distance* from the voter's ideal point. This choice may be justified in one-dimensional models but quickly becomes very cumbersome for spaces with higher dimensionality. Therefore, I will specify

[4]See for example, Coughlin[2]

[5]Of course, this argument would lost most of its appeal in a multi-party model. But that needs not be a concern right now.

[6]The reason for which I claimed earlier that the boundedness of the space is not essential is because computer simulations of the model have up to now shown that there is no difference in the results when one assumes that the voters' ideal points are distributed on an unbounded space for a fairly large family of distributions (including, of course, the bivariate normal). This is due to a fact that I will state here without proof: the contour curves of the probability of victory function, within the relevant range of the unit box, behave exactly the same for these more complicated distributions as for the uniform one.

the voters' behavior according to a different model of discrete choice which, it should be noticed, yields the *exact same results* as the Euclidean model in the one-dimensional case.

The model is the one proposed by Amos Tversky to analyze discrete choices of consumers, called Elimination by Aspects (EBA). Not only is this a plausible decision rule, used by individuals in experiments, but also there is a huge gain in mathematical tractability when we adopt it, as opposed to the most common logistic model. EBA is, for all practical purpose, a lexicographic ordering where the priorities themselves are random. In this case, for example, if a voter is facing two alternative platforms, she will choose the characteristic of the platform she will use as a "first decimal place" in the lexicographic ordering (X or Y), according to a lottery with probabilities $(\beta, 1-\beta)$[7]. Therefore, the parameter β represents the relative salience of the issues for the voters. Once a characteristic has been chosen, the voter will proceed with the ranking of the alternatives. At the individual level, this means that for every pair of platforms L and R, a voter will choose one of them with probabilities 0, β, $1-\beta$ or 1. Now, given that there is a continuum of voters, the law of large numbers will ensure that these individual probabilities can be translated into actual vote shares.

More precisely, if we use the subindexes L, R to denote the platforms proposed by the Left and Right parties, then we can specify the votes garnered by each platform in this way: for two given platforms $(x_L, y_L); (x_R, y_R)$, we can partition the set of voters into four groups:

1. $\{(i,j) | \mid x_i - x_L \mid < \mid x_i - x_R \mid \wedge \mid y_j - y_L \mid < \mid y_j - y_R \mid\}$
2. $\{(i,j) | \mid x_i - x_L \mid < \mid x_i - x_R \mid \wedge \mid y_j - y_L \mid > \mid y_j - y_R \mid\}$
3. $\{(i,j) | \mid x_i - x_L \mid > \mid x_i - x_R \mid \wedge \mid y_j - y_L \mid < \mid y_j - y_R \mid\}$
4. $\{(i,j) | \mid x_i - x_L \mid > \mid x_i - x_R \mid \wedge \mid y_j - y_L \mid > \mid y_j - y_R \mid\}$

The typical member of group 1 will vote for L with probability 1. Thus, 1 will also be the share of voters in group 1 that vote for L. Likewise, the typical voter of group 2 will

[7] A good exposition of Tversky's model can be found in Anderson et al. [2].

vote for L with probability β. This gives a share of β voters within this group that support L. Along the same lines, the support for party L in groups 3 and 4 will be $1-\beta$ and 0 respectively. (Dimension-by-dimension ties occur with probability 0.)

Geometrically speaking, these four sets are rectangles within the unit box with one of their vertices at $(\overline{x}, \overline{y}) = (\frac{x_L+x_R}{2}, \frac{y_L+y_R}{2})$ (see Figure 2.1). Of course, not only the size of the rectangles will depend upon the parties' platforms but also the same will be true for their location. This means that we have to split the analysis in several cases. Here I will confine myself to the two most relevant cases. The other ones will be perfectly analogous but their intuitive appeal is much weaker. The two cases I will consider are:

1. $x_L < x_R$ and $y_L < y_R$
2. $x_L < x_R$ and $y_L > y_R$

As it turns out, given that under the uniform distribution areas translate directly into volumes, the expected vote of the Left party, π_L, becomes rather simple in both cases:

For Case 1:

$$\begin{aligned}\pi_L &= \overline{x}\overline{y} + \beta\overline{x}(1-\overline{y}) + (1-\beta)(1-\overline{x})\overline{y} && (2.1)\\ &= \beta\overline{x} + (1-\beta)\overline{y} && (2.2)\\ &= \beta\frac{x_L+x_R}{2} + (1-\beta)\frac{y_L+y_R}{2} && (2.3)\end{aligned}$$

For Case 2:

$$\begin{aligned}\pi_L &= \beta\overline{x}\overline{y} + \overline{x}(1-\overline{y}) + (1-\beta)(1-\overline{x})(1-\overline{y}) && (2.4)\\ &= \beta\overline{x} + (1-\beta)(1-\overline{y}) && (2.5)\\ &= \beta\frac{x_L+x_R}{2} + (1-\beta)\frac{2-y_L-y_R}{2} && (2.6)\end{aligned}$$

This completes the characterization of the expected vote function. Its importance resides

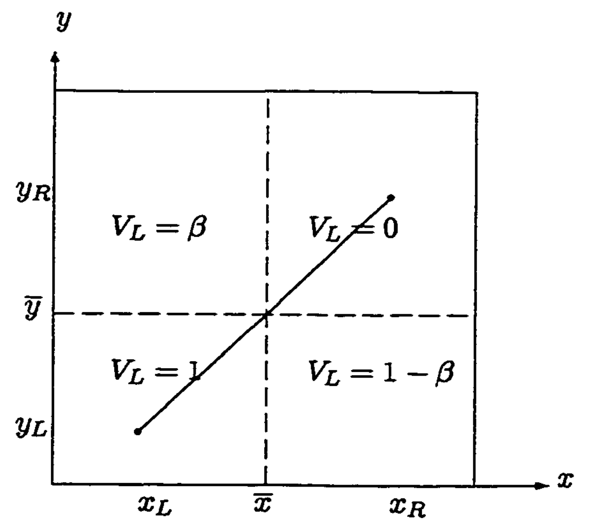

Figure 2.1: Vote-shares for Party L in the policy space.

on the fact that it *is* the pay-off function for the opportunists of each party. (Of course, while Left opportunists' want to maximize π_L, those of the Right want to minimize it.)

2.4.2 The Set of Equilibria

We are now interested in the contract locus between militants and opportunists since all the PUNEa of the game will belong to it. In particular, a platform belongs to this locus if the set of platforms preferred to it by the militants to is disjoint from the set of platforms whose probability of victory is higher than that of this platform. Therefore, the pair of platforms (x_L, y_L, x_R, y_R) is a PUNE iff $\forall d_L, d_R \in \Re^2$:

$$\nabla_L U_{00}(L) \cdot d_L > 0, \nabla_L U_{01}(L) \cdot d_L > 0 \;\; \Rightarrow \tag{2.7}$$

$$\nabla_L \pi(L, R) \cdot d_L < 0 \tag{2.8}$$

$$\nabla_R U_{10}(R) \cdot d_R > 0, \nabla_R U_{11}(R) \cdot d_R > 0 \;\; \Rightarrow \tag{2.9}$$

$$\nabla_R \pi(L, R) \cdot d_R > 0 \tag{2.10}$$

By Farkas' Lemma this implies that there exist constants $\alpha_{00}, \alpha_{01}, \alpha_{10}, \alpha_{11} > 0$ such that:

$$\begin{aligned} \alpha_{00}\nabla U_{00} + \alpha_{01}\nabla U_{01} &= -\nabla_L \pi(L,R) && (2.11) \\ \alpha_{10}\nabla U_{10} + \alpha_{11}\nabla U_{11} &= \nabla_R \pi(L,R) && (2.12) \end{aligned}$$

Finally, "absorbing" constants, this becomes (for Case 1):

$$\begin{aligned} x_L &= \frac{\beta}{\alpha_{00}+\alpha_{01}} && (2.13) \\ y_L &= \frac{1-\beta+\alpha_{01}}{\alpha_{00}+\alpha_{01}} && (2.14) \\ x_R &= 1-\frac{\beta}{\alpha_{10}+\alpha_{11}} && (2.15) \\ y_R &= \frac{\alpha_{11}-1+\beta}{\alpha_{10}+\alpha_{11}} && (2.16) \end{aligned}$$

Thus, any selection of two points from the surfaces described by these equations, that respects the constraint $x_L < x_R, y_L < y_R$ is a PUNE.

It is easy to verify that for Case 2 the equations that characterize the PUNE are very similar, namely:

$$\begin{aligned} x_L &= \frac{\beta}{\alpha_{00}+\alpha_{01}} && (2.17) \\ y_L &= \frac{\alpha_{01}-1+\beta}{\alpha_{00}+\alpha_{01}} && (2.18) \\ x_R &= 1-\frac{\beta}{\alpha_{10}+\alpha_{11}} && (2.19) \\ y_R &= \frac{1-\beta+\alpha_{11}}{\alpha_{10}+\alpha_{11}} && (2.20) \end{aligned}$$

Once again subject to the caveat that the two points selected respect the constraints.

Thus, we can see that this setting is such that the set of PUNEa is very easy to characterize and, hopefully, to analyze. Similar calculations to the ones I just showed, give the set of PUNEa for the alignment in which factions with common positions on the Y dimension group together in parties[8]

If we denote by C and D the parties that compete under this alignment, the following equations characterize the sets from which we can select any arbitrary PUNE:

If $x_C < x_D, y_C < y_D$:

$$\begin{aligned} x_C &= \frac{\beta + \alpha_{01}}{\alpha_{00} + \alpha_{01}} && (2.21) \\ y_C &= \frac{1-\beta}{\alpha_{00} + \alpha_{01}} && (2.22) \\ x_D &= \frac{\alpha_{11} - \beta}{\alpha_{10} + \alpha_{11}} && (2.23) \\ y_D &= 1 - \frac{1-\beta}{\alpha_{10} + \alpha_{11}} && (2.24) \end{aligned}$$

If $x_C > x_D, y_C < y_D$:

$$\begin{aligned} x_C &= \frac{\alpha_{01} - \beta}{\alpha_{00} + \alpha_{01}} && (2.25) \\ y_C &= \frac{1-\beta}{\alpha_{00} + \alpha_{01}} && (2.26) \\ x_D &= \frac{\beta + \alpha_{11}}{\alpha_{10} + \alpha_{11}} && (2.27) \\ y_D &= 1 - \frac{1-\beta}{\alpha_{10} + \alpha_{11}} && (2.28) \end{aligned}$$

This completes the description of the PUNEa in this setting. The next step is, then, to use these results as inputs for the analysis of the first stage, the topic we now turn to.

[8]In this case, the partition will be: $\{(0,0);(1,0)\}, \{(0,1);(1,1)\}$. So the militants' ideal points will have extreme locations along the Y dimension and a weighted average of the factions' ideal locations along the X dimension.

2.4.3 The Systemic Alignment of Parties

Up to my knowledge, the problem of how do factions choose their partners when deciding to form a party has not yet being studied formally. This means that we will have to borrow a solution concept from other areas of decision theory. I believe that the best candidate for such a concept is the *one-sided stable matching* used in the literature of operations research. While *two-sided matching* models have already become part and parcel of economics (doubtlessly, thanks to the extensive work of Alvin Roth[9], one-sided matching has barely been noticed in the profession in spite of the extensive literature already existing on it[10]. Let's then rehearse briefly the structure of the problem and the correspondent solution concept. A one-sided matching model (a.k.a. the "roommate problem" after its most common real-life application) is defined by a finite set of agents, each one endowed with a preference profile that ranks the remaining members of the set[11]. A *matching* is a partition of the agents' set in pairs. Therefore, the problem consists in finding a matching that is not "too much" at variance with the agents' preferences.. A way to think about this is that if a matching is "too bad", the agents will undo it by a series of bilateral contracts. How, then, to define a "bad matching"? The key concept is somehow implied by what will happen afterwards: we are interested in matchings such that the agents have no incentive to undo them via bilateral contracting. This is the requirement of *stability* common in the literature of this type of problems. A matching is defined as stable if *either each agent is paired with its preferred option or its preferred options are matched with agents they rank higher than it.* Let us note, however, one disturbing fact: matching models are essentially discrete. Given that their basic input is made up of preference profiles over finite sets (viz. the possible "roommates"), we cannot represent the individual decision problems, conveniently, in terms of first-order conditions as done in most of economic analysis. This will result in an impossibility of obtaining crisp, analytical results so that we need to rely

[9]Roth and Sotomayor [12] is still the *summa* of this branch.

[10]See for example, Abeledo and Rothblum [1]

[11]This is the major difference with two-sided matching problem (or "stable marriages problem"). In the latter the set of agents is divided in two subsets so that an individual of one subset has preferences defined only over the members of the other subset. Hence the nickname

both on geometric intuition and computer simulations.

Woefully, this is not all the price that we have to pay for the complexities of this problem. We still need to introduce further simplifications. In particular, a major simplification is dictated by the assumption made at the beginning about the alignments that would be considered. On purely intuitive grounds I imposed the constraint that for two factions to belong in the same party, they must share the same view on one dimension. This restriction simplifies the analysis in two ways: first, it reduces the amount of cases we have to consider to two possible alignments. On the other hand, it helps us bypass a problem that plagues some one-sided matching models. Unlike their two-sided counterparts, one-sided matching problems sometimes fail to have stable solutions. One of the most common causes of this is when one individual is ranked worst by all the rest. So, by assuming away the possibility of these unseemly coalitions, we make sure that for each faction the worst possible partner is different (viz. the one at the opposite corner of the box) and then, the non-existence problem disappears. Quite the opposite, the problem we face in this particular model is that for some preference profiles, *both* alignments are stable. This will happen either when there is indifference in some crucial profile (not all indifferences will precipitate such result) or when, with strict preferences, these "cycle" around the ideal points so that each faction is most preferred by one of its neighbors but has the other neighbor as its best option. (A full discussion of all the cases can be found in the respective Appendix A.1.) In general, we will be facing three possible situations: either there is a unique stable alignment, or there are several, in which case the prevailing party system will be determined more by accident or history than by the strategic calculations of the factions, or, as said before, no stable solution will exist and the party system will be in constant flux unless or until a structural change in the exogenous parameters pushes it out of that unstable region. However, once again, this third option is ruled out by the specification of preferences I have adopted.

It is now time to state in an operational manner the question that gives origin to this paper. In political science, for the most part, party alignments are considered as products of "sociological factors". In the categories I have been using here, one could translate this conjecture into saying that the voters' preferences are the ones that ultimately determine the party alignment. So, within the framework of this model, one is led to ask to what extent the prevailing party system is responsive to the voters' preferences (here parameterized by β). In other words, is it true that the parties will always compete along the dimensions perceived as more salient by the voters?

The procedure followed here to answer that question is the following: were we to know which is the *unique* PUNE that will obtain in each of the two alternative electoral subgames that can be played in the model (i.e., the Left-Right game and the Up-Down game), each faction would be able to decide which alignment is best for it. That is, it would be easy to calculate the preference ranking in which each faction places the other ones. Once this ranking is known, it is straightforward to find which is the stable solution of the corresponding matching problem. This solution (or solutions, if there is non-uniqueness) would then be the party alignment.

To fix ideas, let's introduce a further piece of notation:

$$\begin{aligned}
\Delta_{00} &= U_{00}(x_L, y_L) - U_{00}(x_C, y_C) \\
\Delta_{01} &= U_{01}(x_L, y_L) - U_{00}(x_D, y_D) \\
\Delta_{10} &= U_{10}(x_R, y_R) - U_{00}(x_C, y_C) \\
\Delta_{11} &= U_{11}(x_R, y_R) - U_{00}(x_D, y_D)
\end{aligned}$$

where, as before, subindexes L, R denote one alignment (Left-Right) and C, D the other (Up-Down). The preferences of the factions will be dictated by the *sign* of their respective Δ function. Thus, for example, if all the Δ's are positive, it will mean that the only stable alignment is the Left-Right one: it will match each faction with its preferred partner so that no one has incentives to undo it through bilateral agreements. The factions will form parties $\{(0,0);(0,1)\}$ and $\{(1,0);(1,1)\}$ which is to say that they will prefer to cooperate with partners with which they share a common view over the Left-Right dimension *in spite* *of their differences* along the Up-Down dimension.

However, this step presupposes that the equilibrium of each electoral subgame is actually known to all the factions. Regretfully, the concept of PUNE generates a continuum of equilibria for any electoral game. This means that exercises of comparative statics need to rely on a further closure of the system or on simulations of the model. There is a difficulty already mentioned in passing which greatly complicates matters. Refinements that rely on completing the preference ordering for the factions in a party promptly lead to non-existence of equilibria. At this point, the question of how to close the system so as to produce a unique equilibrium (or at least multiple, isolated equilibria) remains open.

But this does not mean that there is no meaningful conclusion we can draw about the problem at hand. In particular, the parameters that characterize each PUNE have a very natural interpretation: for each faction, the corresponding constant that goes into the application of Farkas' lemma indicates the *actual* level curve reached at the particular equilibrium considered; the higher this constant, the higher the pay-off obtained by that faction. Formally, the bargaining power of each faction is represented by the corresponding coefficient α in Eqns. 2.13 through 2.28. The larger the coefficient accompanying a utility function's gradient, the closer the party's platform will be to the respective faction's ideal point. In other words, the parameters that single out one equilibrium also inform us about the *bargaining power* of the militant factions, vis-a-vis each other and the militants. Whatever the internal process of conflict adjudication that a party chooses, the values of these constants provide a clear indication of who prevailed and to what extent. So, in what follows, I will consider these parameters as representing the structure of intra-party struggle (or *party structure* for short). To each vector of constants in $\Re^4$ corresponds a specific party structure.

Following this interpretation, there are some basic results about the effect of voters' preferences over the behavior of the party systems that do not require an specific refinement, that is, they hold true for any set of parameters that yields equilibria within the constraints of the policy space.

Proposition 1 *Given an alignment along dimension j, and a fixed set of parameters, the parties' platforms will* diverge *along that dimension as its salience becomes* lower

The proof of this statement is trivial: it is enough to verify that

$$\frac{\partial x_L}{\partial \beta} > 0, \frac{\partial x_R}{\partial \beta} < 0, \frac{\partial y_C}{\partial \beta} < 0, \frac{\partial y_D}{\partial \beta} > 0$$

There is a clear intuition for this. When an issue is not too salient for the voters, the opportunists do not see their probability of victory decrease too much by taking an extreme stance on it even if that may possibly alienate votes. Therefore, the opportunists can use this issue as a "bone" to throw at the militants in order to gain their acquiescence, while seeking for a more moderate position on the issue that will sway more strongly the voters.

In itself, the following result is not particularly useful, but it helps to characterize the behavior of the party alignments. Its proof is essentially geometric and is relegated to the Appendix:

Proposition 2 *Represent the factional utility functions as* indirect functions $U_{ij}(\vec{\alpha}, \beta)$. *Then, the difference function Δ_{ij} fulfills the* single-crossing property, *that is, there exists a $\tilde{\beta}$ such that, if for any $\beta < \tilde{\beta}, \Delta_{ij}(\vec{\alpha}, \beta) < 0$ (resp. > 0), then for any $\beta > \tilde{\beta}, \Delta_{ij}(\vec{\alpha}, \beta) > 0$ (resp. < 0).*

There are two major implications of this result. First, it means that, given a party structure, as the relative salience of the issues change, we can expect to observe *at most* four "switches" in the factional preferences, one for each faction. This places a tight bound on the amount of realignments that can be observed as the issue salience varies. Regretfully, the bound is not as tight as one could desire. In particular, if we expect to obtain a clean result in which the party alignment moves *once* from Left-Right to Up-Down as the salience of the Y issue increases, then this upper bound is disappointing. It does not rule out parameter combinations where *two* realignments occur. Take for example a party structure such that, when $\beta = 0$, the signs of the Δs are: $\Delta_{00} > 0, \Delta_{01} > 0, \Delta_{10} < 0, \Delta_{11} < 0$ and their respective crossing points are such that $0 < \tilde{\beta}_{00} < \tilde{\beta}_{11} < \tilde{\beta}_{10} < 1 < \tilde{\beta}_{01}$. In this example, as β rises from 0 to 1 two realignments occur: one at $\tilde{\beta}_{00}$ where $(0,0)$ and $(1,0)$ now form a stable matching as opposed to the one formed by $(0,0),(0,1)$ for values below that one, and a second realignment at $\tilde{\beta}_{1,0}$, from which point onwards the stable matching is, once again formed along the Left-Right dimension. Examples like this cannot be ruled out on *a priori* grounds and, moreover, turn up quite often in computer simulations.

The second implication is more useful and ties directly with the main result. The single-crossing property will turn out to be useful when analyzing how responsive is the party alignment to the changes in preferences. Analytical expressions for the changes in the party alignments as a function of β and the party structure are impossible to obtain. However, there is one result of comparative statics that can still be derived without further specifications of the alignments.

If we want to know how *stable* is a party alignment, we need to know how often it will change as the voters' preferences change. On the other hand, those alignment changes occur in a discrete manner: for one of them to take place a *necessary* condition is that a "crossing" occurs in one of the Δ functions. Those crossings, we know, are sparsely distributed over the interval $0 \leq \beta \leq 1$: actually there are *at most* four of them (one per each faction).

So, we can perform this exercise: given two different party *structures* (as defined by the relative bargaining power of the factions), is there any sense in which we can say that one of them generates a party system more stable than the other? Clearly, if the underlying Δ functions of one of them yields less crossings than those of the other one, we can conclude that the party alignments are more stable in the former case.

Although it is infeasible to compare all possible party structures, there is one comparison for which there is a clear answer. Consider two party structures such that the relative bargaining power of the *militant* factions is the same but that differ only in their bargaining power vis-a-vis their respective opportunist factions.

Formally, this is the same as saying that if $\vec{\alpha}, \vec{\alpha'}$ are the two structures to be compared, then there exists a constant $k > 0$, such that $\vec{\alpha} = k\vec{\alpha'}$. So, if $k > 1$, then in the first party structure the militants wield more power than in the second. With this framework in hand the following result is easy to prove.

Proposition 3 *For two different party structures $\vec{\alpha}, \vec{\alpha'}$ such that $\vec{\alpha} = k\vec{\alpha'}$, for $k > 1$, the number of crossings of the different functions $\Delta_{ij}(\vec{\alpha'}, \beta)$ is (weakly) lower than the number of crossings for $\Delta_{ij}(\vec{\alpha}, \beta)$.*

Once again, an algebraic proof would be extremely cumbersome. (A geometric proof is available from the author.) Here I limit myself to an intuition of the proof. If, for example, in Case 1 above (Eqns. 2.13, 2.14), we write the L platforms as

$$x_L = \frac{\beta}{k(\alpha_{00} + \alpha_{01})}$$
$$y_L = \frac{1 - \beta + k\alpha_{01}}{k(\alpha_{00} + \alpha_{01})}$$

it is obvious that the absolute value of both $\frac{\partial x_L}{\partial \beta}, \frac{\partial y_L}{\partial \beta}$ are *decreasing* functions of k. That is, as β varies between 0 and 1, the larger the k, the less will be the resulting variation in the party's platform. So, if the equilibrium platforms are less responsive to changes in β, one can only expect that the overall alignment will also be less responsive to those same changes. To fix ideas, as $k \to \infty, (x_L, y_L) \to (0,0)$ regardless of β, that is, the L platform becomes a constant. As the same occurs to all other platforms, the functions Δ all become constants so that there is a unique party alignment throughout all the interval of possible values for salience. Conceptually, what this result says is that the more the parties are controlled by their militant factions, the more stable the party alignments will be, that is, the more impervious they are to changes in the relative salience of the issues.

An old puzzle in the field of comparative politics is why the US, being one of the first industrialized nations, did not become a polity marked by a partisan struggle between capital and labor to an extent anywhere close to that of the other European industrialized democracies. A common explanation has been that there were many other sources of heterogeneity in the US electorate so that this political cleavage could not become active (see, for example, Burnham [3]). There might well be something to this explanation. In fact, at the firm level, industrialists were perfectly aware of its cogency as is witnessed by the fact that, early in the history of the US labor movement, employers would occasionally use blacks and immigrants to break strikes and weaken the unions, thus activating non-economic splits that could override the class divisions.

But the model presented here suggests that this explanation may not be sufficient. If the question is why did one specific dimension failed to last as a dividing line in a polity, it does not suffice to point at the bewildering amount of other possible dimensions. In principle, a line of conflict can become politicized and remain such in spite of wide changes in its salience. To that effect, the Lipset-Rokkan hypothesis of "alignment freezing" is illustrative. In the framework of this model, another ingredient needs to be present: the incentives of the political factions to actually politicize one issue or the other. All else being equal, the more powerful the "electoral" motive is within parties, the more erratic will be their platform choices, as the relative salience of the issues change and, thus, the more fickle will be the prevailing alignments. The fact, recognized long ago, that American parties were the first ones to become "rational-electoral" parties, remarkably independent from other front organizations like unions, churches, etc. (at least by European standards) lends credence to this hypothesis.

If this is an accurate way of thinking about party alignments, it may also contribute to explain why the European party systems are "thawing" precisely at a point at which the grip held over the parties by their "hard-liners" weakens and they become, so to speak, "more American". However, one should not read too much into this type of analysis. There are always puzzling cases of politicization of social conflicts that are not easily amenable to this type of analysis. As Cox [8] rightly points out, the differences between the Swedish minority and the rest of the population in Finland pales compared to the race-gaps in the US. But in the former case there is a Party of the Swedish People ... On the other hand, the recent trend in American politics towards divided government and "de-alignment" remains beyond the reach of this model.

2.5 Concluding Remarks

The main thrust of the paper can be summarized as follows: in order to analyze how responsive are the party systems in a polity to the changes in the electorate's preferences, I develop a model of electoral competition in which the parties are considered as coalitions, instead of as unitary actors. These coalitions can, therefore, form or dissolve according to the forecasts their members do about their prospects in an election. The incentives that each faction in a coalition has to join it or defect it are analyzed by means of a one-sided matching model. The main result obtained is that the more powerful the militant factions (ideologically oriented) are vis-a-vis the opportunist factions (with a purely electoral motive), the more will the alignment remain impervious to the day-to-day changes of the relative salience of the issues among the voters.

Appealing as the result may seem, there are two major limitations that require further inquiry. First, the predictive power of the model would benefit greatly from a possible refinement of the equilibrium concept used. In the absence of one, we need to rely only on the most general results and on comparative statics of quite limited reach. Second, the process of party alignments is better understood as a dynamic process in which the electoral viability of the parties affect the long-run incentives of its different factions. That this model has remained static is, clearly, a void that calls for attention. On the other hand, it may speak in favor of the model the fact that, for all these shortcomings, it can still capture *some* aspects of the formation of party systems.

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Chapter 3

Legislatures and Political Parties: Endogenous Policy with Strategic Voters

3.1 Introduction

Democratic polities differ in the degree of legislative initiative allowed by their constitutional arrangements. While in some regimes (henceforth called open-rule regimes), the legislature is free to amend any legislative proposal submitted to its consideration, in others (closed-rule regimes), its agenda is limited so that, by and large, it can only vote on the platforms proposed by the political parties. The United States, Great Britain in the period prior to the First Reform Act (1832), and the Fourth Republic in France are well-known cases of open-rule regimes whereas contemporary Great Britain, the Fifth French Republic, Japan and some "strong" presidential regimes, especially in Latin America, constitute examples of democracies akin to the closed-rule model. Intuitively, these two types of system imply different ways of determining the "popular will." Open-rule regimes rest on the assumption that the legislature is the arena where the preferences of the citizenry are aggregated. Local constituencies elect representatives expected to voice their voters' grievances in the forum provided by the legislature. In contrast, in closed-rule regimes such aggregation of individual preferences takes place at the general election, mediated by the political parties. Once the masses have spoken, giving the majority to a party or set of parties, it is the legislature's task to implement their will in as faithful and diligent a manner as possible-hence the constraints placed over its agenda. In this case, were the legislature to introduce amendments of its own over the agenda proposed by the parties it would somehow distort the mandate received.

Given this characterization of the two models of democracy, we should expect them to lead to different allocations of political power and, hence to different policy outcomes. After all, in most democracies, the major parties are nationally-based coalitions of citizens (with regional parties playing a subsidiary role). Therefore, whether the legislature responds to constituencies or to parties is far from an immaterial consideration. The goal of this paper is to give a rigorous treatment to this insight.

To that end, I will develop a model of two abstract polities, identical in all respects except in the degree to which they allow legislative initiative, and then characterize when and how their policy outcomes will differ. A thorough comparison of the two systems requires an analysis of elections (contested by parties) *and* legislation. For the most part, the existing literature on endogenous economic policy has focused either on electoral competition among parties or on legislative decision-making but not on a unified model of both. Examples of party-based analyses are Alesina [1], on macroeconomic policy, Alesina and Tabellini [2] and Persson and Svensson [19] on public debt, Persson and Tabellini [20] on fiscal policy in a common market and Roemer [22, 6] on the democratic class struggle. Legislature-based models are used in Baron and Ferejohn [4] and Weingast, Shepsle and Johnsen [27] to analyze the geographic distribution of costs and benefits of public projects. A skeptical view of party-based models (at least of their relevance for the US) is proposed by Krehbiel [11, 12].

There are other examples of related literature as well, although there are some important ways in which this paper departs from them. A formal comparative perspective on the role of legislatures and parties can be found in Persson, Roland and Tabellini [18]. Diermeier and Feddersen [9] study also legislative institutions and how they affect partisan behavior in the legislature. However, their focus concentrates on one particular institutional setting (the confidence vote in parliamentary regimes). None of these two papers considers the electoral origins of the legislature.

It may be argued that a polity's constitutional framework, far from being a *datum* that affects the choices of the political parties, is in fact the endogenous result of the interaction of those same parties. But in order to understand the endogenous formation of constitutional arrangements, there is prior step that needs to be taken: examining the impact of such arrangements on the opportunities available to the political parties. This is the task undertaken here.

An important feature of the present model is its extensive use of sophisticated voting. The voters use their knowledge about the constitutional setting to assess the real impact of different candidates on final policy outcomes. The idea of combining strategic voting with explicit institutional settings is also present in recent work by Austen-Smith [3]. There are, however, a few differences. First, the present model introduces electoral uncertainty, which will prove to have important implications. Second, Austen-Smith assigns a crucial role to *party constitutions*, that is, social choice rules that generate the parties' platforms from their candidates' strategies, and which will dictate the policy of the elected legislature. As he rightly states, this framework is more reasonable the more the parties can be thought of as controlling the legislature. In contrast, a major goal of this paper is to show how *legislative institutions* can shape policy outcomes, even keeping constant the internal structure of the parties. A central argument of the paper is that this interaction between the political institutions and the electorate's strategic behavior can explain some of the most crucial patterns of legislative behavior.

First, the model predicts that the voting positions observed in the legislature will be correlated with the party labels. In other words, the legislators casting the most 'rightist' votes will systematically belong to the 'Rightist' party while the 'leftist' legislators will belong to the 'Leftist' party. This is such a widely observed pattern in so many different legislatures that we tend to take it for granted. However, it is surprising that many standard models of open-rule regimes that rely on the sincere voting assumption, consistently *fail* to predict this same pattern. In fact, according to these models, extremist legislators should have *equal* chances of belonging to any party.

The present model also takes a step further, generating testable theories as to how such correlation will vary as the distribution of voters' preferences change in a given polity. In this sense, the model is relevant to the literature on quantitative studies of legislative behavior. (McCarty, Poole and Rosenthal [16], Collie and Mason [6] and Smith [25] are examples from this literature which focus on the US Congress.)

In this paper, parties control resources vital for the electoral viability of the candidates. This would suggest that they can use such resources to influence the legislators' behavior regardless of the constitutional setting. However, the second result of this paper shows that decision-making rules that allow for legislative initiative deprive political parties of most (though not all) of their influence over policy outcomes. In this case, the policy will be determined by the median *district* since it is the district that will return the pivotal legislator. In turn, the third result proves that without legislative initiative, the parties propose divergent platforms and that the victorious party is fully able of implementing its platform as policy. Intuitively, legislative initiative allows for the individual members of the legislature to undo the partisan arrangements that may have been made during the electoral phase of the political process.

For decades, comparative political economy has highlighted the extent to which the expansion of franchise in many countries led to the formation of national, class-based parties (e.g. Labor in Britain).[1] However, the US, allegedly the first country to attain virtually universal franchise, followed a very different path. The results of this model suggest the following hypothesis: while Britain, at the time of expanding the franchise, had already completed the transition to a closed-rule regime, the US remained (as it still is today) an open-rule polity. In the light of the findings of this paper, cross-district, class-based coalitions in the US were bound to be much less succesful than what they eventually became in Britain; under an open rule the political parties, for all their control over their candidates,

[1]A list of studies of this nature would constitute a paper in itself. Lipset and Rokkan [13] is one of the classical references. A more recent example is Luebbert [14].

cannot prevent the policy results from gravitating toward the median districts' preferred outcomes.

A fourth result from this model says that the degree to which the constitutional framework affects policy outcomes depends upon the level of intra-district homogeneity of the voters' preferences. In a closed-rule polity, the more homogeneous the preferences *within* the districts, the more the parties' platforms will converge to the median voter of the median district, and hence, the more the policy outcome will resemble the one that would obtain under an open-rule regime. This raises a host of relevant questions about policies that modify the distribution and sorting of citizens across districts. Districts' boundaries and sizes may be changed by legislative fiat, but there are also other, deeper economic forces at work in this regard, like migration. Once the model is developed and analyzed, I will argue in the concluding section that this fact has implications for the way we think of endogenous economic policies in federalized systems.

A widely held view attributes the different legislative patterns observed here to different degrees of "party strength," where the latter is interpreted as the ability of parties to coerce their members' vote in the legislature. Here this is not the case. In this paper, the organizational prerrogatives of the parties vis-a-vis their caucuses remain identical across models. Furthermore, the legislature votes following strict majority rule. I believe this approach, on top of being parsimonious, has the advantage of making explicit what is meant by party strength. In fact, the present model suggests that we ought to distinguish between "system-wide" sources of party strength, emanating from the constitutional allocation of decision-making power and "party-specific" sources, that depend on the internal organization of each party.

The paper is organized as follows: Section 3.2 presents the basic setup and notation of the model. Section 3.3 calculates the equilibrium of the model under an open-rule legislative institution, while Section 3.4 does the same for the closed-rule case. Section 3.5 studies the impact of intra-district homogeneity on the policy outcomes of the model. Finally, section 4.5 summarizes the main results, offers some concluding remarks and discusses possible extensions. Lengthy proofs are relegated to the Appendix.

3.2 Basic Setting, Definitions and Notation

3.2.1 Institutions

A set of citizens is partitioned into N districts, where N is an odd number. The subindex A will be used to denote a generic district while the subindex M will refer to a special district (the "median district") to be defined below; I will make explicit use of the district subindex only when needed to avoid confusion. There is one policy issue common to all districts so that the policy space is represented by $X \subseteq \Re$. Each district elects a *legislator*. All the legislators convene in a single body (the *legislature*) and they choose the policy to be implemented by *majority rule*. I will consider two alternative agenda-setting rules later.

3.2.2 Agents

There are three groups of agents: citizens, (local) candidates, and (national) parties. These have single-peaked preferences over policy outcomes. For convenience, these will be represented by a Euclidean utility function though none of the essential arguments depend on this specific functional form. The ideal policy point of a citizen will then be a sufficient description of his type. To simplify notation, if a citizen's ideal point is x_i, his type is will be written as i. The policy preferences of a type i citizen are described by the function u_i where:

$$u_i(x) = -(x_i - x)^2.$$

The same is true of the parties. In particular, there are two parties: Left and Right, and τ_L and τ_R denote their respective ideal points. The Left's policy preferences will be labeled u_L while u_R will correspond to the Right's policy preferences. Summarizing:

$$u_L(x) = -(\tau_L - x)^2,$$

$$u_R(x) = -(\tau_R - x)^2.$$

Without loss of generality, and to ease visualization, $\tau_L < \tau_R$, that is, the Left party is actually "to the left of" the Right party.

The candidates' objective is to maximize their probability of victory. It is premature to explain how this probability is calculated since we need some other ingredients of the model. In every district there are two candidates, one for each national party. Thus, l and r will represent the Left's and Right's candidates respectively (with their respective district subindex when needed).

3.2.3 Stages of the Game and Strategies

The game consists of the following stages:

Convention Stage: Parties L and R each choose a point in the policy space, respectively called x_L and x_R, which will be their *national platforms.*

Campaign Stage: The $2N$ candidates choose points in $\Re$ which will represent their *local platforms.* The local platform of candidates l, r in district A will be denoted x_{lA} and x_{rA} respectively. These local platforms become common knowledge for the national parties but are not yet disclosed to the voters.

Endorsement Stage: The parties decide, for each district, whether or not to field their respective candidate. If party L nominates a candidate in district A, I will write $e_{LA} = 1$; otherwise, $e_{LA} = 0$. Analogous definitions hold for party R.

Electoral Stage: The local platforms of those candidates actually fielded are revealed to the citizens. A state ω is chosen by Nature. Within each district, a group of voters (whose distribution is governed by ω) is selected from the pool of citizens and elections take place. So, the electoral outcome is uncertain.

Legislative Stage: The victorious candidates become legislators and convene to decide the policy. Each legislator votes according to Euclidean preferences whose ideal point is the *local platform* she announced in the campaign stage. From now on, x_A will denote the platform of district A's *legislator*, i.e. the platform of the victorious candidate ($x_A \in \{x_{lA}, x_{rA}\}$). The set of ideal points of a legislature will be $\mathbf{x} = \{x_1, \ldots, x_N\}$. The final policy outcome, x^*, depends on *all* the strategies chosen in the other stages.

Strictly speaking:

$$x^* = x^*(x_L, x_R, \{x_{lA}\}_{A\in\{1,\ldots,N\}}, \{x_{rA}\}_{A\in\{1,\ldots,N\}}, \{e_{lA}\}_{A\in\{1,\ldots,N\}}, \{e_{lA}\}_{A\in\{1,\ldots,N\}}).$$

To simplify notation, when referring to x^*, I will drop the arguments not directly relevant to the claim being made.

3.2.4 Legislative Rules

It is important to keep in mind that the legislature in this model always uses majority rule to arrive to any decision. What changes between one institutional setting and the other is the way the agenda is shaped. In one case, the *open-rule* system, the legislature is allowed to introduce any kind of amendment to the legislative proposals it receives. In contrast, under the *closed-rule* regime, the agenda is composed of only two possible alternatives: the national platforms x_L, x_R proposed by the parties. No further amendment is possible. Formally, let a denote the agenda- that is, the set of policy alternatives admissible to be considered by the legislature. The two possible settings are then:

1. Open-rule legislature: $\mathbf{a}_O = \Re$.

2. Closed-rule legislature: $\mathbf{a}_C = \{x_L, x_R\}$.

3.2.5 Auxiliary Assumptions

The basic setup just described needs to be complemented by some extra assumptions, specifically with regard to *a)* the distribution of voters within districts, *b)* how is electoral uncertainty introduced, *c)* what the role of endorsements is, and *d)* how the probability of victory is calculated within the model.

Electoral Districts There is a continuum of citizens in every district. Further, across districts the distribution of citizens' preferences varies. Thus, letting G_A be the cumulative distribution of citizens' ideal points in district A, with cumulative distribution G_B for district B, then in general, $G_A(x) \neq G_B(x)$ for any x. Define the location of the median citizen in district A by μ_A (i.e. $G_A(\mu_A) = 1/2$). Without loss of generality, label districts so that $\mu_1 < \ldots < \mu_M < \ldots < \mu_N$, where $M \equiv \frac{N+1}{2}$; therefore, district M is the *median district*. Furthermore, the parties' ideal points will be assumed to bound the ideal point of the median citizen in the median district: $\tau_L < \mu_M < \tau_R$. This is a plausible assumption. Without it, the "left" party would be to the right of the majority of districts (or the "right" party would be to the left of them).

Electoral Uncertainty The approach used here to introduce electoral uncertainty is the one common in models of probabilistic voting (see, for instance, the work of Calvert [5], Coughlin [2], Wittman [28] and Roemer [22], among others). Let $\omega \in [0, 1]$ be a state chosen by Nature at election day with a cumulative distribution $F(\omega)$. After that draw, in every district a sample of *voters* is selected from the set of citizens. The actual location of the *median voter* in each district depends upon the state ω selected. Formally, denoting by i_{mA} the median voter of district A, then $i_{mA}(\omega)$ is a strictly increasing function of ω. Thus, for any location of the median voter, we can retrieve the value of the state with the inverse function i_m^{-1}. Furthermore, I will make the following assumption about the behavior of i_m: In every district, for any two intervals of types $[i_0, i_1], [i'_0, i'_1]$,

$$G(i_1) - G(i_0) \geq G(i_1') - G(i_0') \iff F(i_m^{-1}(i_1)) - F(i_m^{-1}(i_0)) \geq F(i_m^{-1}(i_1')) - F(i_m^{-1}(i_0')).$$

This assumption means that the mapping from states to locations of the median voter is such that as more citizens are located in certain interval, the more likely it is that the median voter belongs to that same interval. It is a plausible condition on the responsiveness of the process generating the median voter location to the actual distribution of preferences in the electorate.

From this assumption, we obtain a simple but useful lemma. Its proof is straightforward and is peripheral to the main point of the paper, so I omit it:

Lemma 1 *For every district A, the location of its median voter in the median state ω_M, defined by $F(\omega_M) = 1/2$, is equal to the location of its median citizen, that is: $\forall A, F(i_m^{-1}(\mu_A)) = 1/2$.*

I will also assume that in every district, $G(\tau_L) > 0$ and $G(\tau_R) < 1$. This assumption implies that both parties have some support (no matter how small) in every district.

Since there is electoral uncertainty, the parties cannot know what the exact effect of supporting their candidates will be. The policy outcome is decided by a legislature whose composition is unknown at the endorsement stage. From now on, I will denote by $p(\mathbf{x})$ the probability that legislature $\mathbf{x}$ is elected. So, both parties and voters evaluate a candidate by the expected utility they obtain from her being elected. For example, for party L, the expected utility derived from electing candidate l in district A is given by

$$E(u_L(x_{lA})) = \sum_{\mathbf{x}_{-A}} u_L(x^*(\mathbf{x}_{-A}, x_{lA}))p(\mathbf{x}_{-A}),$$

where a similar definition holds for party R and for any given citizen of type i.

Endorsements In this model, parties command resources essential for the electoral viability of the candidates. The following assumptions capture the basic elements of the relationship between parties and candidates:

- If a candidate is not endorsed by her party, her probability of victory is 0.
- If, at the end of the endorsement stage, a candidate turns out to run unopposed, the platform she will disclose is the ideal point of her party. That is: $e_{LA} = 1, e_{RA} = 0 \Rightarrow x_{lA} = \tau_L; e_{LA} = 0, e_{RA} = 1 \Rightarrow x_{rA} = \tau_R$.
- If both parties refuse to field a candidate in a district, this district's legislator will be selected at random from the citizens so that the distribution of the would-be legislator's preferences is the same as the distribution of citizens' preferences.

The first assumption implicitly denies the possibility of independent candidates, that is, candidates that run without the support of one of the national parties. The second assumption specifies how the candidate breaks ties between strategies if her victory is assured. The third assumption amounts to claim that the only case in which independent candidates have any viability occurs when the two major parties fail to field a candidate of their own. This is consistent with the basic outlook adopted in the first assumption.

Probability of Victory When both candidates in a district are endorsed, their probability of victory is a function of the platforms proposed. From now on, $\pi_A(x_l, x_r)$ will denote the probability of victory of candidate l in district A, given that the platforms are x_l, x_r. By the same token, $1 - \pi_A(x_l, x_r)$ will denote r's probability of victory.

Legislators vote according to their platform regardless of party labels. If both candidates in a district choose the same platform, their votes in the legislature will be exactly the same so their impact on the policy outcome chosen will also be the same. Hence, voters will be indifferent between them and their vote will be determined by the toss of a fair coin. In any other case, we need to know the location of the type of voter indifferent between the two candidates. This location depends upon the legislative rule employed, so, for the time being, we derive the general form of the probability of victory, filling the details once we enter the discussion of each institutional setting.

Under sincere voting, single-peakedness has the convenient implication of partitioning the electorate into two *convex* sets of types: one supporting the Left candidate and the other supporting the Right. The following lemma ensures that the same is true under strategic voting.

Lemma 2 (Single-Crossing Property of Voter's Preferences) *Denote by i^* the voter indifferent between two candidates in any district. If $x^*(\mathbf{x}_{-A}, x_A)$ is monotonic non-decreasing in x_A, then $x_{lA} < x_{rA}$ implies that:*

- $\forall i < i^*, x_{lA} \succ_i x_{rA}$ *and*
- $\forall i > i^*, x_{lA} \prec_i x_{rA}$

Proof: See Appendix B.1.1.

Without loss of generality, we describe how the probability of victory is determined for the case in which the types of voters who vote for l are lower (to the left of) those who vote for r. In this case, l will win her district's election if the type of the median voter is lower than i^*, or

$$\Pr(i_m < i^*) = \Pr(\omega : i_m(\omega) < i^*) \equiv F(i_m^{-1}(i^*)).$$

Putting all these results together, we obtain:

$$\pi(x_l, x_r) = \begin{cases} F(i_m^{-1}(i^*)) & \text{if } \ x_l < x_r \\ 1/2 & \text{if } \ x_l = x_r \\ 1 - F(i_m^{-1}(i^*)) & \text{if } \ x_l > x_r \end{cases}$$

Notice that the data of a political system are given by the makeup of the electorate (as defined by the distributions G_A and the median-voter generating processes i_{mA}), the legislative institution and the ideal points of the parties. We can now bring together all these elements of the model in the following definition:

Definition 1 *A **polity** is a collection* $\mathcal{P} = \langle \{G_A\}_{A\in\{1,\ldots,N\}}, \{i_{mA}\}_{A\in\{1,\ldots,N\}}, \mathrm{a}, \tau_L, \tau_R\rangle$.

Now we need to specify the solution concept to be used. Verbally, it is simply a restatement of the usual conditions for a Nash equilibrium of a game, that is, that all the strategies are chosen optimally taking as given all the remaining strategies of the other players.

Definition 2 *Given a polity* $\mathcal{P}$, *a **political (Nash) equilibrium** is a collection of strategies* $\langle x_L^*, x_R^*, \{x_{lA}^*\}_{A\in\{1,\ldots,N\}}, \{x_{rA}^*\}_{A\in\{1,\ldots,N\}}, \{e_{LA}^*\}_{A\in\{1,\ldots,N\}}, \{e_{RA}^*\}_{A\in\{1,\ldots,N\}}\rangle$ *such that:*

- $x_L^* = \arg\max_{x_L\in\Re} E(u_L(x^*(x_L, x_R^*)))$ *with a perfectly analogous condition for* x_R^*.
- *For each* A, $x_{lA}^* = \arg\max_{x_{lA}\in\Re} \pi(x_{lA}, x_{rA}^*)$, $x_{rA}^* = \arg\min_{x_{rA}\in\Re} \pi(x_{lA}^*, x_{rA})$
- $e_{LA}^* = \arg\max_{e_{LA}\in\{0,1\}} E(u_L(x^*(x_L^*, x_R^*, \{e_{LB}^*\}_{B\neq A}, \{e_{RA}^*\}_{A\in\{1,\ldots,N\}})))$ *with an analogous condition for* e_{RA}^*.

Under this definition, the parties choose their national platforms x_L^* and x_R^* so as to maximize the expected utility derived from the (uncertain) electoral outcome. The same is true of their endorsement strategies $\{e_{LA}^*\_A \in \{1,\ldots,N\}\}$ and $\{e_{LA}^*\_A \in \{1,\ldots,N\}\}$. Notice that the endorsement strategy of a party in district A depends not only on the other party's endorsement strategy in that same district but also on the strategies chosen by *both* parties in *all* the remaining districts. In turn, the candidates choose local platforms that maximize their probability of victory in their respective district.

Our next task is to solve for the political equilibria of the two types of political systems considered here: those with open rules and closed rules. That is the goal of the next sections. To distinguish between the two types of model, I will call $\mathcal{P}_O$ an open-rule polity and $\mathcal{P}_C$ a closed-rule polity.

The following result, pertaining to the endorsement strategies, is common to both models. It says that in any district, in equilibrium both candidates are endorsed and the parties are not indifferent between the platform of their candidate and yielding that district to the other candidate running on the ideal point of her party:

Lemma 3 *For any polity $\mathcal{P}$, in a political equilibrium the endorsement strategies and the local platforms are such that, for all districts A*

1. $e_{LA} = e_{RA} = 1$
2. $E(u_L(x^*(\mathbf{x}_{-A}, x_{lA}))) > E(u_L(x^*(\mathbf{x}_{-A}, \tau_R)))$ *and*
3. $E(u_R(x^*(\mathbf{x}_{-A}, x_{rA}))) > E(u_R(x^*(\mathbf{x}_{-A}, \tau_L)))$

Proof: If inequalities 2 and 3 do not hold, then the parties are indifferent between endorsing a candidate or just yielding that district to the other party. So, any randomization between, say, $e_{LA} = 1$ and $e_{LA} = 0$ is consistent with sub-game perfection. From the point of view of the candidates, securing endorsement is a dominant strategy. Therefore, if (say) x_{lA} is such that 2 does not hold, party L has the credible threat of choosing $e_{LA} = 0$ so that x_{lA} is not optimal for the candidate. If 2 and 3 hold, then it is easy to verify that $e_{LA} = 0, e_{RA} = 1$ is not an equilibrium, because party L could increase its pay-off by choosing $e_{LA} = 1$. But, neither is $e_{LA} = e_{RA} = 0$ an equilibrium. In that case, A's representative will be choosen from G_A so that there is a positive probability that it will be of type τ_R or τ_L. So, L can secure that she is *not* of type τ_R by simply deviating to $e_{LA} = 1$.

3.3 The Open-Rule Model

Given that this is a multi-stage game, the natural solution concept is that of backward induction. In this section, I will solve the game for the case of open-rule institutions using the customary procedure of solving first for the last stages and working up the decision tree until reaching the first stage.

Legislative Stage As has been stressed repeatedly throughout the paper, this is the only stage in which there is some structural difference between both types of political systems. As said before, under an open rule, the set of alternatives to be voted upon is not constrained in any way. Any member of the legislature can introduce, if she so desires, a new point in the agenda (an *amendment*) to be considered against whatever may be the *status-quo* at the time.

Ideally, a full description of the legislative institution used in each case would include the extensive form of the game underlying the policy-making process. However, there is no need for such detail: the reduced-form spelled out here is enough. It is a well-known fact that under the conditions specified in this definition, the policy outcome is the ideal point of the median legislator. To make this precise, the operator $m(\cdot)$ will denote the median of a set of ideal points. Hence, under an open rule, $x^* = m(\mathbf{x})$.

Electoral Stage The median operator is monotonic non-decreasing with respect to the legislators' platforms. Therefore, Lemma 2 ensures that under strategic voting every district's electorate is partitioned into two convex sets of types, each supporting a different candidate. Thus, if $x_{lA} < x_{rA}$, there will exist a type i^* such that all voters left of i^* will vote for l and, likewise, all voters to the right of i^* will vote for r.

Endorsement Stage As described in Lemma 3, all candidates secure endorsement by choosing platforms such that their parties are not indifferent between fielding them and yielding that district to the other party.

Campaign Stage There is a continuum of equilibria for the local platforms. However, they all share some very important properties. The following theorem describes this set of equilibria. Due to its length, its proof can be found in Appendix B.1.2.

Theorem 1 *Let* $\mathcal{P} = \langle \{G_A\}_{A\in\{1,\dots,N\}}, \{i_{mA}\}_{A\in\{1,\dots,N\}}, \mathbf{a}_O, \tau_L, \tau_R \rangle$ *be an open-rule polity and* ϵ *an arbitrarily small constant such that* $0 < \epsilon < \min[\mu_{M+1} - \mu_M, \mu_M - \mu_{M-1}]$. *Then, in any equilibrium the local platforms of the candidates are such that:*

- $x_{lA} < \mu_M - \epsilon, x_{rA} = \mu_M + \epsilon$ *if* $A < M$,
- $x_{lM} = \mu_M = x_{rM}$
- $x_{lA} = \mu_M - \epsilon, x_{rA} > \mu_M + \epsilon$ *if* $A > M$

Intuitively, the endorsement constraints force the candidates to take stances that, if elected, will lead to a different median legislator at least for some possible electoral outcomes. However, since this differentiation is costly for the candidates in terms of their probability of victory, they try to minimize it by remaining as close as possible to their districts' medians.

The following result is valuable for what follows. Its proof is trivial and will be omitted:

Corollary 1 *For all possible legislatures, the location of the median legislator* $m(\mathbf{x}) \in \{\mu_M - \epsilon, \mu_M, \mu_M + \epsilon\}$.

The second corollary of this theorem, although straightforward to prove, is very important: it establishes that the political equilibria of this model generate a correlation between voting stances and party labels in the possible legislatures. In other words,

Corollary 2 *For an open-rule polity* $\mathcal{P}_O$*, given that a legislator's platform is to the left of that of the median legislator, her probability of belonging to the Left party is greater than her probability of belonging to the Right party: that is*

$$\Pr(x_A = x_{lA} \mid x_A < m(\mathbf{x})) > \Pr(x_A = x_{rA} \mid x_A < m(\mathbf{x}))$$

and, likewise,

$$\Pr(x_A = x_{lA} \mid x_A > m(\mathbf{x})) < \Pr(x_A = x_{rA} \mid x_A > m(\mathbf{x}))$$

Proof: I will consider only the first statement, the second being proven by identical arguments. If we can prove that $\Pr(x_A = x_{lA}, x_A > m(\mathbf{x})) > \Pr(x_A = x_{rA}, x_A > m(\mathbf{x}))$, the result follows. Now:

$$\begin{aligned}
&\Pr(x_A = x_{lA}, x_A > m(\mathbf{x})) \\
&\quad = \Pr(x_A = x_{lA}, x_A < \mu_M - \epsilon) + \Pr(x_A = x_{lA}, \mu_M - \epsilon \leq x_A < \mu_M, m(\mathbf{x}) \geq \mu_M) + \\
&\qquad \Pr(x_A = x_{lA}, \mu_M \leq x_A < \mu_M + \epsilon, m(\mathbf{x}) = \mu_M + \epsilon) \\
&\quad \leq \Pr(x_{M-1} = x_{lM-1}) + \Pr(x_N = x_{lN}, m(\mathbf{x}) \geq \mu_M) + \Pr(x_M = x_{lM}, m(\mathbf{x}) = \mu_M + \epsilon)
\end{aligned}$$

where the last inequality comes from the fact that the only l candidates that adopt platforms $< \mu_M - \epsilon$ are those running in districts $1, \ldots, M-1$ while the l candidates with platforms $> \mu_M - \epsilon$ run in districts $M+1, \ldots, N$. Applying a similar reasoning to the other expression (that for the Right) we conclude that:

$$\begin{aligned}
&\Pr(x_A = x_{rA}, x_A > m(\mathbf{x})) \\
&\quad = \Pr(x_A = x_{rA}, x_A < \mu_M - \epsilon) + \Pr(x_A = x_{rA}, \mu_M - \epsilon \leq x_A < \mu_M, m(\mathbf{x}) \geq \mu_M) + \\
&\qquad \Pr(x_A = x_{rA}, \mu_M \leq x_A < \mu_M + \epsilon, m(\mathbf{x}) = \mu_M + \epsilon) \\
&\quad = \Pr(x_M = x_{rM}, m(\mathbf{x}) = \mu_M + \epsilon)
\end{aligned}$$

because M is the only district where an r candidate chooses a platform $< \mu_M + \epsilon$. Since in district M, both candidates have a probability of victory of $1/2$, the claim follows.

The reason this corollary is so important is as follows: a legislature generated by this model will be such that the legislators to the left of the median member are more likely to be from the L party while those to the right are more likely to be from the R party. The most 'rightist' legislator of the L party will be close to the center and the same is true for the most 'leftist' legislator of the R party. In other words, there will be a correlation between voting stances and party membership a pattern displayed by many real-life legislatures.

Space considerations rule out a detailed discussion, but it is important to realize how this result would break down if we dropped any of the crucial assumptions.

Case 1. No endorsement constraints, sincere voting: Here both candidates in every district will choose the platform that maximizes their probability of victory, not unlike the standard Downsian model. In that case, the probability of victory for all candidates is $1/2$. This implies that the most leftist legislator is equally likely to be from the Left or from the Right. Notice that this overtly wrong prediction comes from the *standard* median-voter model when applied to the multi-district case.

Case 2. Endorsement constraints, sincere voting: Under these assumptions, the candidates still need to differentiate themselves from their rivals so as to secure endorsement. But, sincere voting means that voters fail to see through the legislative decision-making process. Therefore, the "majority" candidates (that is, the Left candidates in districts $A \leq M - 1$ and the Right candidates in districts $A \geq M + 1$) benefit from choosing platforms as close as possible to those of their rivals as far as is consistent with securing endorsement, *even if this does not affect* the possible median locations in the legislature. In that case, all the candidates will choose platforms infinitesimally close to μ_M. Needless to say, this is another implausible prediction.

Convention Stage As seen in Corollary 4.3, in equilibrium under an open rule the median legislator, and hence the policy outcome, is infinitesimally close to μ_M and, furthermore, her location is independent of the party platforms.[2] Therefore, the parties' pay-offs, are also independent of their national platforms. The optimal national platforms are then arbitrary. All the elements of this model can be put together in the following theorem, whose proof is already contained in the preceding arguments:

Theorem 2 (Legislative Government under Open Rule) *Let $\mathcal{P}$ be a polity with the legislative institution of open rule prevailing and ϵ a vanishingly small constant* $0 < \epsilon <$

[2]Alternatively, if we think of endorsements as costly for the parties, then the value of ϵ would be the one necessary for them to recover such cost.

$\min[\mu_{M+1} - \mu_M, \mu_M - \mu_{M-1}]$. *Then, in all the political Nash equilibria:*

- $e^*_{L,A} = e^*_{R,A} = 1 \ \ \forall A \in \{1, \dots, N\}$
- *The local platforms are as described in Theorem 1*

and the policy outcome x^* *is independent of* x_L, x_R *and is such that* $x^* \in \{\mu_M - \epsilon, \mu_M, \mu_M + \epsilon\}$.

Remark: Notice that under an open rule, although national platforms are irrelevant, parties themselves are not. In fact, it is thanks to their endorsement prerrogatives that the policy outcome is not fixed at μ_M. Were we to use a sincere voting model with no endorsement constraints, the policy outcome would be entirely fixed at μ_M. In that case, it would be hard to explain why political parties come to exist in the first place. But this model shows that, although limited, there is a role for parties: they have the possibility of fielding minority candidates that, if elected (no matter how unlikely this may be) will actually modify to some extent the legislature's choice.

3.4 The Closed-Rule Model

Legislative Stage As said in the introduction, the main difference between open and closed rules as interpreted here is the fact that in the latter the agenda is entirely dictated by the parties' platforms. Therefore, the policy outcome is the party platform which obtains the simple majority in the legislature:

$$x^* = \begin{cases} x_L & \text{if} \quad \#\{A : u_A(x_L) > u_A(x_R)\} > \#\{A : u_A(x_L) < u_A(x_R)\} \\ x_R & \text{if} \quad \#\{A : u_A(x_L) > u_A(x_R)\} > \#\{A : u_A(x_L) < u_A(x_R)\} \end{cases}$$

where, u_A stands for the utility function of district A's legislator which is, as said before, the utility function whose ideal point is the platform she announced as a candidate.

Electoral Stage Under a closed rule, the only relevant feature of the candidates the voters care about is the agenda point they will support (x_L or x_R) if elected. The candidates' specific platforms become *irrelevant*. Unlike in standard spatial models, here $u_i(x_l) > u_i(x_r)$ is not a sufficient condition for voter i to vote for l. In fact, in spite of this difference in utility, if both candidates would support the same platform in the legislature, if elected, all the voters will be indifferent between them and the probability of victory for both candidates will be 1/2. However, the inequality is still a necessary condition. Therefore, if the candidates vote differently in the legislature (e.g. $u_l(x_L) > u_l(x_R), u_r(x_L) < u_r(x_R)$), the indifferent type in every district is: $i^* = \frac{x_L+x_R}{2}$. So:

$$\pi_A(x_l, x_r) = \begin{cases} F(i_{mA}^{-1})\left(\frac{x_L+x_R}{2}\right) & \text{if } x_l < x_r \\ 1/2 & \text{if } x_l = x_r \\ 1 - F(i_{mA}^{-1})\left(\frac{x_L+x_R}{2}\right) & \text{if } x_l > x_r \end{cases}$$

Endorsement Stage Once again, each candidate is endorsed and the platform she chooses is such that the parties are not indifferent between fielding her or allowing the other party's candidate to win her district.

Campaign Stage Once again, securing endorsement is a dominant strategy for the candidates. Therefore, from the analysis of the endorsement stage, we conclude that for the candidates to achieve this, they need to propose local platforms that support their respective parties. This can be stated as the following lemma:

Lemma 4 *For a polity $\mathcal{P} = \langle \{G_A\}_{A\in\{1,\dots,N\}}, \{i_{m,A}\}_{A\in\{1,\dots,N\}}, a_C, \tau_L, \tau_R\rangle$, in any equilibrium, the local platforms of the candidates are such that*

$$| x_{lA} - x_L^* | < | x_{lA} - x_R^* |, | x_{rA} - x_L^* | > | x_{rA} - x_R^* | \quad \forall A$$

Convention Stage Lemma 4 is crucial to the analysis of the convention stage under a closed rule. The first thing to notice is that in this model, unlike what happens in the open rule model, there is one sense in which a local candidate is really a *party's* candidate: if elected she will support her party's national platform and that is the end of it. In the closed rule model, a party's victory implies more than simply obtaining a majority of dubious relevance in the legislature. Here victory means actually getting to implement its national platform.

There is a second implication of this result. From the point of view of a national party, the general election resembles strikingly a ***single-district*** election under majority rule, where each district represents an individual vote. Call $\Pi(x_L, x_R)$ the probability that party L wins a majority of seats in the legislature, i.e. $\Pi(x_L, x_R) = \Pr(\#\{A : u_A(x_L) > u_A(x_R)\} > \{A : u_A(x_R) > u_A(x_L)\})$. Therefore, each party's pay-off becomes:

$$E(u_L(x_L)) = u_L(x_L)\Pi(x_L, x_R) + u_L(x_R)(1 - \Pi(x_L, x_R))$$

$$E(u_R(x_R)) = u_R(x_L)\Pi(x_L, x_R) + u_R(x_R)(1 - \Pi(x_L, x_R))$$

Notice that this is exactly the same pay-off function usually assumed for parties with policy preferences in one-district elections. This enables us to use a well-known result of spatial competition which will form part of the following theorem characterizing the equilibrium under closed rules.

Theorem 3 (Party Government under Closed Rules) *Let $\mathcal{P}$ be a polity with the legislative institution of closed rule. Then, in all the political Nash equilibria*

- $e^*_{LA} = e^*_{RA} = 1 \;\; \forall A \in \{1, \dots, N\}$
- $x^*_{lA} < \frac{x^*_L + x^*_R}{2}, x_{rA} > \frac{x^*_L + x^*_R}{2} \;\; \forall A \in \{1, \dots, N\}$
- $x^*_L < x^*_R$

and the policy outcome x^* is the national platform that obtains the majority of the legislature: $x^* \in \{x^*_L, x^*_R\}$.

Proof: The first two claims of the theorem have already been proven. The third claim requires a somewhat lengthy proof, to be found in Appendix B.1.3

Remark: According to this result, the candidates will virtually become "employees" of their parties, with the task of supporting it in the legislature. This coincides with the way individual legislators are normally thought of in closed-rule regimes.

3.5 Legislative Outcomes and the Electorate's Structure: Comparative Analysis

The two main theorems show how the legislative institutions contribute to shape the policy outcomes in a democracy. However, their relative importance actually depends on the makeup of the electorate. Is there any special feature of the citizenry that will dictate the actual performance of the legislative institutions in shaping the policy outcomes? The main result of this section is that there is one: the degree of *intradistrict heterogeneity.*

Before going into the main results, it is worth making precise what it will mean formally for a polity to experience an increase in intra-district homogeneity. To that end, I will generate a sequence of polities for which the electorate becomes increasingly homogeneous at the district level. The first step is, for every district A, choose a sequence of intervals $\{[\underline{\mu}, \overline{\mu}]\}_{A,n}$ of citizens' types with the following properties:

- $\underline{\mu}_{A,n} < \mu_A < \overline{\mu}_{A,n}$ $\forall n$ i.e. each of the n intervals bounds the median citizen's type in district A.
- $\forall m > n, \overline{\mu}_{A,m} - \underline{\mu}_{A,m} \leq \overline{\mu}_{A,n} - \underline{\mu}_{A,n}$ i.e. as we go further in the sequence, the intervals narrow.
- $\lim_{n\to\infty} \overline{\mu}_{A,n} - \underline{\mu}_{A,n} = 0$ i.e. in the limit, the intervals become arbitrarily small.

For every district A, consider a sequence of probability measures $\{G_A\}_n$ such that:

- $G_{A,n+1}(\mu_A) = G_{A,n}(\mu_A) = 1/2, \quad \forall n, A.$
- $G_{A,n+1}$ assigns at least as much probability as $G_{A,n}$ to every subinterval in $[\underline{\mu}, \overline{\mu}]_{A,n}$.
- $G_{A,n+1}$ assigns no more probability than $G_{A,n}$ to every subinterval in $(-\infty, \underline{\mu}]_{A,n}$.
- $G_{A,n+1}$ assigns no more probability than $G_{A,n}$ to every subinterval in $[\overline{\mu}, \infty)_{A,n}$.

Therefore, $G_{A,n+1}$ is a *median-preserving risk reduction* (henceforth m.p.r.r) of $G_{A,n}$. (This definition is analogous to the definition of mean-preserving risk reduction given in Machina and Pratt [15].) A sequence of measures thus constructed will be said to describe a sequence of polities with ***increasing intradistrict homogeneity.***

What are the effects of a change of intradistrict homogeneity in the two cases? The easiest case to analyze is that of the open rule. Since the only determinant of the policy outcome is the location of the median citizen in the median district, we know that the legislation implemented will not change. However, the correlation between party and ideology will increase as the districts become more homogeneous. This is a testable implication of the model, related to the current studies on legislative polarization in countries like the US. This is the claim of the following lemma:

Lemma 5 *Let $\mathcal{P}_{O,n}$ be a sequence of polities under an open rule with increasing intradistrict homogeneity. Then, the probability of a legislator to the left of the median belonging to the Left party goes to one. The same is true about the probability of a legislator to the right of the medin belonging to the Right party. Formally,* $\lim_{n\to\infty} \Pr_n(x_A = x_{lA} \mid x_A < m(\mathbf{x})) = 1$. *Analogously,* $\lim_{n\to\infty} \Pr_n(x_A = x_{rA} \mid x_A > m(\mathbf{x})) = 1$.

Proof: There are only two types of candidates running on local platforms in the interval $[\mu_M - \epsilon, \mu_M + \epsilon]$: l candidates running in districts $A > M$ and r candidates running in districts $A < M$. Focusing only on the first group, their probability of victory is $\pi_A(x_l, x_r) \approx F(i_{mA}^{-1}(\mu_M))$.

Since the sequence of polities is such that intradistrict homogeneity is increasing, then $\lim_{n\to\infty} \pi_A(x_l, x_r) = 0$. Therefore, $\lim_{n\to\infty} \Pr_n(\mu_M - \epsilon < x_A < \mu_M + \epsilon) = 0$, and therefore:

$$\begin{aligned}
&\lim_{n\to\infty} \Pr_n(x_A = x_{lA} \mid x_A < m(\mathbf{x})) \\
&= \lim_{n\to\infty} \frac{\Pr_n(x_A = x_{lA}, x_A < \mu_M - \epsilon) + \Pr_n(x_A = x_{lA}, \mu_M - \epsilon < x_A < \mu_M + \epsilon)}{\Pr_n(x_A < \mu_M - \epsilon) + \Pr_n(\mu_M - \epsilon < x_A < m(\mathbf{x}))} \\
&= \lim_{n\to\infty} \frac{\Pr_n(x_A = x_{lA}, x_A < \mu_M - \epsilon)}{\Pr_n(x_A < \mu_M - \epsilon)} \\
&= 1
\end{aligned}$$

A *decrease* in *inter*district homogeneity would have the same effect of raising the correlation between ideology and party in the legislature. This can be seen by fixing μ_M and allowing all the values $\mu_A < \mu_M$ to shift leftwards and all the values $\mu_B > \mu_M$ to shift rightwards as described in the next lemma:

Lemma 6 *Let $\mathcal{P}_O, \mathcal{P}'_O$ be two open rule polities identical in all respect except for:*

- $\mu'_A < \mu_A$ *for all* $A < M$
- $\mu'_M = \mu_M$
- $\mu'_A > \mu_A$ *for all* $A > M$.

Then, $\Pr(x_A = x_{lA} \mid x_A < m(\mathbf{x})) < \Pr(x'_A = x'_{lA} \mid x'_A > m(\mathbf{x}'))$.

Proof: The l candidates in districts $A < M$ in polity $\mathcal{P}'_O$ have a higher probability of victory than in polity $\mathcal{P}_O$. This is due to the fact that the indifferent type i^* remains at the same location in both polities ($\approx \mu_M$) but the distributions G'_A for $A < M$ assign more mass to the interval $(-\infty, \mu_M]$ than the distributions G_A. An analogous argument holds for the probability of victory of the r candidates in districts $A > M$. Since all these are the only candidates that chose platforms out of the interval $[\mu_M - \epsilon, \mu_M + \epsilon]$, the result follows.

Remark: This result is in keeping with findings in American politics about the rise of "partisanship" in Congress, as related to changes in the electorate.[3] As districts become

[3] See, for example, Smith [25].

more different from each other, the legislatures become more polarized. Notice, however, that this does not imply necessarily more extreme policy outcomes. In the example described in the lemma, the median legislator remains in the same location throughout the changes in the electorate, so that the policy choices do not change.

Closed-rule polities respond quite differently to intradistrict homogeneity. In this case, the policy outcome actually changes as a function of changes in the districts' preference distribution. Moreover, as the districts become more homogeneous, the policy outcomes approach what would obtain under an open-rule institution. In other words, the degree to which the constitutional setting affects the policies of a society depend on the way voters are sorted into districts. That is the result of the next theorem. To grasp the intuition behind it, consider what would happen if intradistrict heterogeneity were entirely suppressed, that is, if in each district all the citizens were of the same type and, therefore, the location of the median voter became deterministic. In that case, from the point of view of the parties, their probability of winning any specific seat in the legislature would be either 0 or 1, regardless of the pair of platforms chosen. But that means that for the election as a whole, the probability of victory would also be either 0, 1 or 1/2 (the last of these, only if both propose the same platform). It is a well-known fact that without electoral uncertainty even parties with policy preferences will converge in their platforms. Now, where will this convergence occur? For all intents and purposes, each district becomes, from the point of view of the parties, like a single voter. Therefore, it is not surprising that for each party, the maximin strategy is to propose as a policy the ideal point of *the* voter of the median district. Since this is a zero-sum game, the standard arguments of Downsian convergence apply and hence in equilibrium both parties propose that same platform.

Theorem 4 *Let $\{\mathcal{P}_{C,n}\}$ be a sequence of closed-rule polities with increasing intradistrict homogeneity. Then, the sequence of equilibrium policy outcomes $(x^*_{L,n}, x^*_{R,n})$ is such that $x^*_{L,n} \to \mu_M, x^*_{R,n} \to \mu_M$.*

Remark: Technically, this theorem is the multi-district version of a result obtained previously by Roemer [22]. However, the interpretation given here is different since in this

Table 3.6: Determination of Policy under each Legislative Rule

		Type of Regime	
		Open-Rule	Closed-Rule
***Intra*district Distribution of Preferences**	Heterogeneous	Median District	(National) Parties
	Homogeneous	Median District	Median District

model there is an explicit reason for the reduction of the electoral uncertainty: changes in the population's distribution.

Proof: See Appendix B.1.4

3.6 Concluding Remarks

Before going into a broader discussion of the results of this paper, I will summarize them. The paper argues that constitutional rules, coupled with strategic voting, dictate the balance of power between the political parties and the legislature. The main theorems of the paper are consequences of this basic insight. First, strategic voting generates legislatures in which their members' voting stances are correlated with their party labels. (That is, "leftist" legislators belong to the "Left" party.) In open-rule legislatures, this correlation increases as *intra*district homogeneity increases and *inter*district homogeneity decreases. Second, closed-rule legislatures allow for a determinant role of parties and, moreover, for policy differentiation between them. In contrast, under an open rule, for all their prominence as endorsers of local candidates, the parties cannot prevent the policy outcomes from gravitating toward the median citizen of the median district. Furthermore, under closed-rule legislation, the parties' decisiveness in the policy-making process is undermined as the electorate is sorted into increasingly homogeneous districts. In the limit, if all the districts are populated by voters with identical preferences, then both parties propose the same platform, which coincides, once again, with the median citizen of the median district. In other words, the closed-rule model produces the same policy outcome as the open-rule model. Table 3.6 summarizes this result.

This comparative analysis is relevant for the political economy of federalism. Models of competition between local communities in the provision of public goods (in the tradition begun by Tiebout [26]) generally conclude that fiscal federalism leads to a stratification of agents whereby the population of each community becomes increasingly homogeneous in terms of preferences and income (see for example Epple and Romer [10]). A tacit assumption in the analysis of endogenous economic policies under federalism is that there is some sort of separability between the local and the national components of taxation. According to this view, competition between localities may affect (in fact, is intended to affect) the level of public goods provision and hence the level of taxation within each community, while leaving intact national taxation. This separability has powerful political implications: if it holds, it implies that whatever the distributional consequences of fiscal federalism, governments will still have the possibility of undoing them through national, redistributive taxation. Under this assumption, fiscal federalism should be able to command a wide consensus: individuals of opposing views on income distribution could support it, "saving their energies" for the discussion of distributive taxation.

The model proposed here challenges this separability assumption. It argues that the endogenous choice of national policies depends on the specific way in which voters are sorted into districts. Therefore, it predicts that, in a closed-rule regime, fiscal federalism will have unintended consequences for the endogenous determination of economic policy. As local jurisdictions are allowed to choose different tax schedules and different levels of public goods provision, this generates incentives for the citizens to move to their preferred localities. In other words, the jurisdictions become populated by agents with similar tastes and income. By virtue of Theorem 4, such increase in intradistrict homogeneity brought about by stratification raises the cost the parties would incurr were they to propose extremist platforms. As extremist districts become "safe seats" for the parties, the latter have more incentives to win the centrist districts by converging toward their ideal points. If the relevant policy dimension is economic redistribution (e.g. through taxes) as occurs so often in most modern democracies, this convergence will shape the spectrum of politically viable distributive proposals. As citizens sort themselves into homogeneous districts, a massive process of "spontaneous gerrymandering" is set in motion, enhancing the strategic importance of the median citizens (arguably, the middle class in industrialized democracies), that now live and vote in homogeneous and pivotal districts. Parties cannot afford to ignore these voters.[4]

The characterization of legislative institutions offered here matches observations made in countries that have undergone significant constitutional changes affecting the degree of legislative initiative. The upsurge of party government in England, ushered in by the First Reform Act of 1832, brought an end to what has been called the "Golden Age of the MP." It has been argued (see Cox [8]) that this shift in the balance of power between the parties and Parliament is responsible for the modern pattern of partisan voting both among the electorate and the MP's, just as predicted by this model.

The prediction of correlation between ideology and party in the legislature provides support for the assumption of strategic voting. Usual models of constituency elections, when solved under the assumption of sincere voting, predict that the local candidates will converge to their district's median point so that they should have equal chances of winning. Were this to be true, all the legislators across the ideological spectrum would have a 50/50 chance of belonging to any of the two parties, a prediction consistently falsified by representative bodies in many different contexts.

The results about intradistrict heterogeneity have implications for our understanding of the political effects of structural changes among the electorate and their interaction with constitutional rules. A first area where these results could be put to use is in the current research that looks for connections between the electorate's structure and the parties' presence in the American Congress. Given that the U.S. largely resembles an open-rule regime (as defined in this paper), the hypothesis that increased intradistrict homogeneity and interdistrict heterogeneity increase the party vote in open-rule legislatures is, in principle, testable.

[4]It is important to emphasize that the model's assumption about single-peakedness in voters' preferences is supple enough to accommodate different economic decision problems. Even when single-peakedness does not obtain, problems of distributive taxation can be formulated so that, under some general properties, there is a "natural" ordering of citizens. Thus, the conventional arguments about the decisiveness of the median voter (which is the crucial point here) still hold (see for instance Roberts [21], Meltzer and Richard [17], and Roemer [22]).

Another place where this type of modeling could be employed is in the study of electoral districting and the historic process of franchise expansion. In fact, in any country, these two legal procedures are the most immediate mechanism to alter the composition of the electorate. The conclusions derived from strategic voting could shed light on the impact of such measures on the policy outcomes chosen by democracies under different constitutional frameworks.

Finally, another direction in which this line of argument can be extended lies in the analysis of other types of electoral systems. This paper has focused entirely on single-member district elections. But a fuller understanding of the interaction between legislative institutions and characteristics of the electorate requires the analysis of proportional representation. This I regard as a necessary step in future research.

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Chapter 4

Some Properties of the Probabilistic Voting Model

4.1 Introduction

The purpose of this paper is to discuss some implications of the standard model of probabilistic voting. Two of them place it at odds with the canonical results of the spatial theory of voting in central aspects. The third constitutes a interesting, and somehow plausible, prediction with respect to the patterns of electoral turnout. The paper is, then, organized as follows: Section 4.2 presents briefly the main ingredients of the model, not because of their novelty but as to keep them present for further discussion. Section 4.3 discusses the features of probabilistic voting models in the light of the median voter theorem and Section 4.4 presents its implications for electoral turnout. Section 4.5 offers some final remarks.

4.2 Probabilistic Voting: Basic Elements

The first generation of spatial models of voting (e.g. Downs [3]) assumes that the behavior of each voter is deterministic. That is, when faced with situations identical in all the substantive aspects, the response of the voter will also be identical. If this is so, then the parties can use this knowledge to calculate how their own decisions will affect the voters' and, through these, the parties' electoral performance.

Probabilistic voting was introduced as an attempt to generalize this basic model allowing for some uncertainty. In fact, the underlying decision model postulated by probabilistic voting implies that the voters' decisions may be affected by other factors not taken into account explicitly in the description of the model. Of course, this can be taken as simply an indication that the model has been poorly specified. But this view seems quite extreme. Few students of human behavior would claim to have a description of its causes so rich as to be able to do away with uncertainty. More importantly, the assumption of probabilistic voting implies that, regardless of the decision protocol adopted by the individual voter, and how well the voter himself may know it, the parties, that is, the other main actor of the game who use the voter's decision as an input, cannot observe such protocol and therefore, must allow for some uncertainty in its computations.

In recent years, probabilistic voting has been modelled along the lines employed in economics to analyze problems of discrete choice. This is a natural and salutary step. In fact, I will argue later that one well-known result of the probabilistic voting literatue is not consistent with one of the standard micro models of chcice and hence, the discussion around it should be reopened.

Let's rehearse the standard discrete choice model[1]. To avoid unnecessary duplications, I will introduce it in the context of voting in a very specific environment instead of giving first a general version, to be narrowed down later on. There is a two-party system with parties L and R competing in elections. The policy space is one-dimensional and, hence, represented by the real line. So, $x \in \Re^1$ represents an arbitrary policy position in that space.

Citizens face the problem of deciding for which party to vote given that they propose platforms x_L, x_R. Voters are non-strategic in the sense that they do not take into account the decisions of other voters when making their choices so that, their vote is determined solely by their own assessment of the two parties. Not without circularity, the model assumes that

[1]This exposition follows the one contained in Anderson et al. [1].

a citizen votes for a party (or abstains) if the pay-off he derives from that party (or from abstaining) exceeds that of the other alternatives. To transform this into an operational statement, a further specification of what the pay-off is, is needed. The cornerstone of spatial theory of voting is that this pay-off will depend on the ideological difference between the voter and the parties' proposals. This idea is retained in probabilistic voting only that it is juxtaposed with the possibility of other, non-spatial, factors intervening.

Summarizing, let $u_i(x) = -(x_i - x)^2$ be the ideological distance between voter i, whose most preferred policy is x_i and proposal x. So, denoting by $v_i(x_L)$ the total pay-off a citizen obtains from party L, we write:

$$v_i(x_L) = u_i(x_L) + \epsilon_{Li}$$

where ϵ_L encapsules the non-spatial components of the citizen's assessment of party L.

Since the model allows for abstention, we need an extra piece of notation for that alternative. From now on O will be used to denote abstention so that $v_i(O)$ is, obviously, the utility a citizen obtains from not voting. Since the utility obtained from not voting cannot, by definition, depend on any spatial characteristics of the alternative (there are none), then we simply state that $v_i(O) = \epsilon_{Oi}$.

Calling c_i the citizen's choice, it will be dictated by the following set of inequalities:

$$c_i = \begin{cases} L & \text{if} \quad v_i(L) > v_i(R) \quad \text{and} \quad v_i(L) > v_i(O) \\ R & \text{if} \quad v_i(R) > v_i(L) \quad \text{and} \quad v_i(R) > v_i(O) \\ O & \text{if} \quad v_i(O) > v_i(L) \quad \text{and} \quad v_i(O) > v_i(R) \end{cases}$$

The terms $\epsilon_{Ri}, \epsilon_{Li}$ and ϵ_{Oi} are not observable by the parties. So, the best they can do is to assume some distribution over those terms and use it to infer the probabilities for each possible response. Different distributional assumptions lead to different specifications of the model. It is not uncommon to use the particular assumption that these terms follow a logistic distribution. It is widely perceived as a sensible compromise between the analytical tractability of the uniform distribution and the conceptual plausibility of the normal distribution. (Especially because the behaviors of the logistic and the normal is fairly similar except for the thickness of the tails.) A further simplification can be obtained if we realize that ϵ_{Oi} can simply be absorbed by the other terms. That is, we can interpret $\epsilon_{Li}, \epsilon_{Ri}$ as subuming not only the unobservable utility components of L and R but also how they rank vis-a-vis the alternative of not voting. In this case, ϵ_{Oi} becomes a constant and, moreover, there is no loss of generality in assuming that its value is 0. Under these assumptions, the probability of a citizen with ideal point x_i voting for L becomes:

$$\begin{aligned} P(c_i = L) &= P(u_i(x_L) - u_i(x_R) > \epsilon_{Ri} - \epsilon_{Li}, u_i(x_L) > \epsilon_{Oi} - \epsilon_{Li}) \\ &= \frac{e^{u_i(x_L)}}{1 + e^{u_i(x_L)} + e^{u_i(x_R)}} \end{aligned} \tag{4.1}$$

From now on, I will use the shorthand notation $p_i(L)$ to denote $P(c_i = L)$. In principle, the number of votes each party receives among voters of the same type i is, then, a random variable perhaps with a very unwieldy distribution. However, since, by assumption, voters are non-strategic, their voting decisions are independent from each other. Then, for large electorates, the law of large numbers ensures that the total vote received by party L among type i voters converges in distribution to $p_i(L)$.

Under these conditions, the parties probability of victory is a function of their expected margin of votes over the whole electorate which, in turn, is equal to the weighted average of the type-specific margins, where the weights are given by the relative amount of voters in each type. For the sake of completeness, it is assumed that ties are broken by the toss of a fair coin so that, in that case, the probability of victory of each party is equal to 1/2. Formally, let $f(i)$ be the density of the distribution of types over the policy space. So, if we denote by $\pi(x_L, x_R)$ the probability of victory for party L given the platform choices x_L, x_R, then:

$$\pi(x_L, x_R) = \begin{cases} 1 & \text{if} \quad \int (p_i(L) - p_i(R)) f(i)\, di > 0 \\ 1/2 & \text{if} \quad \int (p_i(L) - p_i(R)) f(i)\, di = 0 \\ 0 & \text{if} \quad \int (p_i(L) - p_i(R)) f(i)\, di < 0. \end{cases} \tag{4.2}$$

This completes the description of the model if the two parties care only about their electoral performance. Throughout the paper I will focus on this formulation.

4.3 The Median Voter Theorem and Probabilistic Voting

Against popular belief, the classical median-voter theorem holds for any one-dimensional distribution of the voters' types whatsoever. In particular, unimodality of the distribution is entirely supefluous to obtain the result of convergence to the median. I will state this result here without proof.

Theorem 5 *Let the pay-off function of parties L, R be defined by $\pi(x_L, x_R), 1 - \pi(x_L, x_R)$ respectively where $\pi(x_L, x_R)$ has the form of Eqn.4.2. Furthermore, let $\epsilon_{Li}, \epsilon_{Ri} = 0$. Then, if x_L^*, x_R^* are the equilibrium platforms, $\pi(x_L^*, x_R^*) = 1/2$ and $x_L^* = x_R^* = \hat{x}$ where $\int_{-\infty}^{\hat{x}} f(i)\, di = 1/2$, for any probability density $f(i)$.*

In an early development of this literature, Enelow and Hinich [4] discovered that, when the assumption of deterministic voters' behavior is dropped, convergence of the two parties will not necessarily occur at the median. Mathematically speaking, $p_i(L) - p_i(R)$, as a function of i, is antisymmetric around $\hat{x} = \frac{x_L + x_R}{2}$, viz. the indifferent type. That is, $p_{\hat{x}-\delta}(L) - p_{\hat{x}-\delta}(R) = p_{\hat{x}+\delta}(R) - p_{\hat{x}+\delta}(L)$. Now, for the integral of an antisymmetric function with respect to an arbitrary probability measure to be equal to 0, which is the necessary condition for an equilibrium, it does not need to be the case that $\hat{x}$ is the median of the distribution. This occurs only if such antisymmetric function takes only two possible constant values k and $-k$ which is exactly what happens in the deterministic model (where $k = 1$).

On the other hand, Enelow and Hinich state a general result of convergence for probabilistic models where convergence occurs at some central position different than the median. Then, to reconcile their finding with empirical observations about divergence of candidates, they appeal to an argument about inflexibilities in the parties' platforms, that lead them to aim toward the centrist position, which is optimal, without being fully able of doing so. More precisely, the parties' positions are seeing as relatively stable through time so that, at the time of a particular election, they can be thought of as fixed. This argument is not fully convicing. In particular, introducing timing considerations in a static model like this, without making explicit the dynamics to which it gives rise, is bound to create confusion.

As it turns out, the convergence result obtained by Enelow and Hinich is not consistent with the probabilistic model presented here. Consider the following example, one of the simplest possible. Let there be an electorate formed by N voters with $N/2$ belonging to type i and $N/2$ belonging to type $-i$. Therefore, the probability of victory of L is:

$$\pi(x_L, x_R) = \begin{cases} 1 & \text{if } (p_i(L) + p_{-i}(L)) - (p_i(R) + p_{-i}(R)) > 0 \\ 1/2 & \text{if } (p_i(L) + p_{-i}(L)) - (p_i(R) + p_{-i}(R)) = 0 \\ 0 & \text{if } (p_i(L) + p_{-i}(L)) - (p_i(R) + p_{-i}(R)) < 0 \end{cases}$$

The total margin of votes for party L is:

$$M(x_L, x_R) = (p_i(L) + p_{-i}(L)) - (p_i(R) + p_{-i}(R)) = \sum_{j \in \{-i, i\}} \frac{e^{u_j(x_L)} - e^{u_j(x_R)}}{1 + e^{u_j(x_L)} + e^{u_j(x_R)}}$$

The first-order conditions for an equilibrium are:

$$
\begin{aligned}
\frac{\partial M}{\partial x_L} &= \sum_{j \in \{-i,i\}} \frac{e^{u_j(x_L)} u_j'(x_L)(1+2e^{u_j(x_R)})}{1+e^{u_j(x_L)}+e^{u_j(x_R)}} = 0 \\
\frac{\partial M}{\partial x_R} &= \sum_{j \in \{-i,i\}} \frac{e^{u_j(x_R)} u_j'(x_R)(1+2e^{u_j(x_L)})}{1+e^{u_j(x_L)}+e^{u_j(x_R)}} = 0
\end{aligned}
$$

Since the game is symmetric, we can focus our attention on symmetric equilibria of the form $x = x_L^* = -x_R^*$. So, the two equations can be merged into one which depends simply on i, $-i$ and x:

$$
\begin{aligned}
e^{u_i(-x)-u_{-i}(-x)} \frac{i+x}{x-i} &= e^{u_i(x)-u_{-i}(x)} \frac{x-i}{-x-i} \\
e^{-4xi} \frac{x+i}{i-x} &= 1
\end{aligned}
$$

Clearly, if $x = 0$, then there is convergence of the parties' platforms in equilibrium. Moreover, $x = 0$ is a solution for this equation. But, since this is simply a necessary condition, this does not imply that $x = 0$ is an equilibrium of the game. In fact, for values of $i > 1$, it is not.

The most rigorous way to prove this is by checking the second-order conditions. Regretfully, they are quite cumbersome so that adducing an example may be a simpler way of showing the same point. Obviously, if $x_L = x_R = 0$, then $M(x_L, x_R) = 0$. But, if $i > 1, M(-i, 0) > 0$ which means that, if $x_R = 0$ there is a profitable deviation for party L.

Does this mean that the Enelow-Hinich result on convergence is wrong? No. The reason for the discrepancy between this example and their claim is that their proof of convergence relies on an additional property of the individual choice probabilities. In particular, EH assume that the type-specific margin $p_i(L) - p_i(R)$ is *separable*, that is, it can be written as $f_i(x_L) - f_i(x_R)$ for some arbitrary function f_i. As they themselves put it, separability ensures that the policy alternatives can be placed in a transitive ranking according to their expected plurality or, to put it in other terms, perhaps more familiar, that there exists one and the same dominant strategy for both parties.

However, there remains a problem. Separability, in the sense that EH give to the word, is *not* a property of the individual choice probabilities in the multinomial logit model typical of discrete choice analysis[2]. As the expression for $p_i(L) - p_i(R)$ shows, both terms depend at the same time on x_L and x_R. In short, EH's result is not incorrect but it is not clearly derived from microfoundations. I will discuss the implications of this in a later section.

4.4 Patterns of Electoral Turnout in Spatial Voting

Arguably, the presence of biases in the pattern of turnout is the major reason to be concerned about abstention. A cynical could claim that, if the profile of voters entirely coincides with that of the citizenry at large, then abstention would be a good thing: the whole businness of electing leaders would be conducted at a lower cost than if everybody voted. Therefore, it is a bit surprising that the rational choice literature on turnout has focused mainly on the *levels* of turnout, rather than in its *patterns*. Here I will present a simple model that displays interesting and testable patterns of turnout.

The following economic environment is fairly standard by now and is borrowed from Roemer [5]. Let the economy consist on a continuum of individuals each one endowed with some income w. There are two goods in the economy: a private good x and a public good G. The preferences of the individuals are represented by:

$$u(x, G) = x + h(G),$$

where h is an increasing, concave function. Income is distributed according to the probability measure $F(w)$ so that, for a given tax rate $0 \leq t \leq 1$, the tax revenue is:

[2]The multinomial logit model displays another type of separability, also known as "Independence of Irrelevant Alternatives". This property refers to the fact that the probability of choosing one element out of a subset does not depend on the utility associated to elements out of that subset.

$$\int tw\, dF(w) = t\mu,$$

where μ is the mean income of the population. If we impose a balanced-budget constraint, then $G = t\mu$ so that the utility of each individual can be expressed as a function of her income and the tax rate:

$$v(w,t) = (1-t)w + h(t\mu).$$

Just as in the previous section, there are two parties L, R that compete for office proposing policy platforms, in this case tax rates. It is easy to verify that this environment is totally in line with the general spatial model of voting spelled out before. To that end, consider when will an individual prefer one platform over the other. If $t_1 < t_2$:

$$v(w,t_2) > v(w,t_2) \iff w > \frac{h(t_2\mu) - h(t_1\mu)}{t_2 - t_1}.$$

This is to say that, although the utility function is not Euclidean as in Section 4.2, it still is monotonically decreasing with the distance between the policy proposal and the individuals ideal point[3]. Therefore, a major simplifying property of spatial models in one-dimension, viz. that each pair of platforms partitions the set of voters in two connected sets, is preserved.

In what follows I will analyze the behavior of electoral turnout as we change the economic fundamentals of the model. To fix ideas, let $h(G) = \log G$ and let w follow a Pareto

[3]In fact, were an individual given the opportunity to decide single-handedly the tax rate of society, she would maximize her indirect utility setting $t^* = h'^{(-1)}\left(\frac{w}{\mu}\right)\frac{1}{\mu}$ where $h'^{(-1)}$ is the inverse of the derivative of h. Since h is concave, h' is monotonically decreasing and, hence, $h'^{(-1)}$ is a monotonically decreasing function of w.

distribution. These two modelling choices are mainly dictated by convenience. Unfortunately, it still remains to be seen if a general theorem could subsume them. Still, to my knowledge, the basic result I will report here also holds for more complicated specifications.

Using the same setup as in previous sections, let $m(w)$ be the margin by which the L vote exceeds the R vote among voters with income w, given tax platforms t_L, t_R, that is:

$$\begin{aligned} m(w, t_L, t_R) &= p_w(L) - p_W(R) \\ &= \frac{\mu(t_L e^{(1-t_L)w} - t_R e^{(1-t_R)w})}{1 + t_L e^{(1-t_L)w} + t_R e^{(1-t_R)w}} \end{aligned}$$

Therefore, the total expected plurality is:

$$M(t_L, t_R) = \int m(w, t_L, t_R)\, dF(w)$$

The first thing to note is that the equilibrium of this game is characterized by full convergence of the parties' platforms. Unfortunately, at this point I have not been able to develop a full proof of this statement. It has been confirmed in simulation after simulation of the model but proving it requires a better account of the distribution of preferences. In fact, in the example of Section refmedian, divergence obtains largely because the distribution of voters' preferences is bimodal. To further complicate matters, notice that in that example, divergence of the platforms did not hold for any two arbitrary locations of the voters. In particular, when the voters are "close enough", the two parties' optimal strategies converge. Therefore, although it is not hard to produce examples in which bimodality of the distribution of voters leads to divergence, it is not the case that unimodality is a necessary condition for convergence. To my knowledge, the characterization of necessary and sufficient conditions for convergence in general environments of probabilistic voting is still an open question in the literature. Therefore, it is not surprising that, given that the environment discussed here is "well-behaved", we can obtain convergence even though there is no self-evident reason for it.

Conjecture 1 *Let $M(t_L, t_R)$ and $-M(t_L, t_R)$ be the objective functions of parties L and R respectively, then, if t_L^*, t_R^* are the equilibrium platforms of the electoral game, then $t_L^* = t_R^*$.*

The results that will be reported shortly (Table 4.1) were obtained in this way: I calculated the (convergent) equilibrium platforms for different specifications of the model where the parameter that is left to vary is the shape parameter of the Pareto distribution. The reason for this is the following: if w follows a Pareto distribution with parameters (a, b) where $a > 0$ is the minimal value w can take and $b > 1$ is the shape parameter, then it is easy to prove that the Gini coefficient G is determined by b in the following manner (see P.K. Sen [7]):

$$G = \frac{1}{2b-1}$$

Of course, b can take any value > 1 but, as we see from the expression for the Gini coefficient, the variation over the interval $(1, 2.5]$ already covers most of the Gini coefficients observable in real life. Therefore, there is little purpose in simulating the model for values of b larger than 2.5.

Several results are worth mentioning: First, abstention is decreasing in income, a readily observable regularity. This is a simple property of the multinomial logit. In a convergent equilibrium, the probability of a voter abstaining is:

$$1 - p_w(L) - p_w(R) = \frac{1}{1 + t_L^* \mu e^{(1-t_L^*)w} + t_R^* \mu e^{(1-t_R^*)w}}$$

which is clearly decreasing in w.

More interestingly, this bias in turnout has a further implication: Just as mentioned before, convergence will not occur at the median. In fact, the equilibrium strategies are the ideal tax of a voter with higher income than the median voter. This leads to what I regard as

Shape Parameter (b)	Gini Coefficient	Equilibrium Platform ($t^* = t^*_L = t^*_R$)	Median-voter Platform
1.1	0.833	0.176	0.533
1.2	0.714	0.21	0.561
1.3	0.625	0.247	0.587
1.4	0.556	0.285	0.61
1.5	0.5	0.321	0.63
1.6	0.455	0.356	0.648
1.7	0.417	0.389	0.665
1.8	0.385	0.419	0.68
1.9	0.357	0.448	0.694
2.0	0.333	0.474	0.707
2.1	0.313	0.498	0.719
2.2	0.294	0.521	0.73
2.3	0.278	0.542	0.74
2.4	0.263	0.561	0.75
2.5	0.25	0.579	0.758

Figure 4.1: Electoral Equilibria of a Taxation Model

the most interesting implication: The bias of the electoral outcome is an *increasing* function of income inequality. As discussed in the next section, if this result obtains in larger classes of economic environments, it has serious implications for the way we think of democratic decision-making.

4.5 Concluding Remarks

In this paper I have developed a standard model of probabilistic voting in order to analyze some of its implications. In particular, several remarkable features emerge. First, the standard results of the spatial theory of voting suffer substantial modifications when the assumption of deterministic voting decisions is dropped. The median voter no longer plays a privileged role to the extent to which the equilibrium platforms may converge somewhere else. Moreover, it is now possible to construct examples in which, for particular distributions of the voters' preferences, convergence no longer holds at all. Furthermore, it is far from obvious which distributional assumptions are necessary to ensure convergence. There are reasons to presume that unimodality is a sufficient condition but even this claim is still not proven. On the other hand, results exist in the literature that prove convergence at equilibrium but, as I said before, they come at the expense of an assumption of separability not borne out by the multinomial logit model.

These features lead to two difficult interpretive questions. First, the qualitative differences between the probabilistic model and its deterministic counterpart put us in front of a dilemma: if taken at face value, the two models cannot be correct at the same time. In fact, for example, if the polarization of voters' preferences is large (i.e. the distribution is bimodal and the two modes are far enough), it is not true that the deterministic model is, as we could naively expect, an approximation of the richer probabilistic specification.

As was discussed in Section 4.3, it is not clear exactly what status should be given to convergence results such as the one obtained by Enelow and Hinich. While the assumption of separability has not yet been derived from first principles, it is fair to add that the multinomial logit model employed here is not without weaknesses. In particular, it is vulnerable to the "red bus-blue bus" problem, that arises when the assumption of independence of the stochastic terms across alternatives does not hold. Coughlin citeCoughlin goes as far as suggesting that this failure should be sufficient to rule out the usage of the multinomial logit model to analyze turnout or elections with more than two candidates. However, this would depend on how the voters perceive the parties vis-a-vis the alternative of abstainig. Certainly, if there is correlation between the utilities of the two parties, then it may be true that the drawbacks of the multinomial logit outweight its simplicity. But, on the other hand, if we are to restore convergence in the Enelow-Hinich framework, we need some other decision-theoretic model of discrete choice in which their type of separability obtaines. It is far from clear what type of model this would be.

Section 4.4 points to another substantive discrepancy between deterministic and probabilistic models. I believe its implications are as much or even more serious as those of the other contrasts pointed out since it bears on a classical question about the effectiveness of democracy in handling conflicts over income inequality. For centuries, political philosophers have wondered whether or not democracy can lead to the expropiation of the rich at the hands of the poor. In fact, early arguments against universal franchise were often couched precisely in this terms: extension of suffrage was seen as a threat to the property rights of society. A more moderate view, but in the same vein, holds that under democracy income inequalities are self-correcting to the extent to which skewed income distributions, that is, distributions in which the median income is below the mean income, will, by the force of the majority (or, the "median voter theorem" in formal terminology) lead to outcomes that implement redistributive measures. In fact, this is what happens in a deterministic voting model. However, the picture changes entirely when we deal with probabilistic models. In the simulation presented above, it is always the case that mean income is above median income, regardless of the shape parameter (except, of course, at $b = \infty$, when the Gini is 0). But, nevertheless, in all the electoral equilibria, the outcomes are biased in favor of voters with income higher than the median. Not only this, but, as said before, the more unequal the distribution, the larger this bias. Under this scenario, inequality is not self-correcting. Unequal societies will tend to perpetuate their inequality by the very nature of their electoral equilibrium. In a nutshell, to the question of "why the poor do not expropriate the rich in a democracy" (a question recently rekindled by Roemer [6]), the probabilistic model offers a terse answer: because the poor do not vote.

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Appendix A

Proofs of Results for Chapter 2

A.1 Deriving Party Alignments form Factions' Preferences

Since one-sided matching models are not part of the economist's staple diet, let's go step by step through the derivation of the solutions.

It will make things easier if we introduce an extra piece of notation. Let's define the following functions:

$$\begin{aligned}
\Delta_{00} &= U_{00}(x_L, y_L) - U_{00}(x_C, y_C) \\
\Delta_{01} &= U_{01}(x_L, y_L) - U_{00}(x_D, y_D) \\
\Delta_{10} &= U_{10}(x_R, y_R) - U_{00}(x_C, y_C) \\
\Delta_{11} &= U_{11}(x_R, y_R) - U_{00}(x_D, y_D)
\end{aligned}$$

So, the Δ's determine how each faction ranks the two possible alignments of the game. They are all defined so that $\Delta > 0$ means that the respective faction prefers the alignment Left-Right over the alignment Down-Up. (Ties are ignored)

The input of any single instance of the one-sided matching problem is fully described by the signs of these functions. Hence, there are 16 possible cases. The solution is then a partition of the set of factions into two pairs so that no two members of the set can, on their own, agree on an option better for both of them. (Notice that this entirely analogue to the definition of the core in a cooperative game.)

Evidently, if *all* the Δ's have the same sign, this will mean that all the factions will prefer the same alignment. For example, if all are positive, the alignment will be $\{(0,0);(0,1)\}$ and $\{(1,0);(1,1)\}$, the Left-Right alignment.

For stability it suffices that *only* one pair is such that its members rank each other best. Therefore, each time that $\Delta_{00} > 0$ and $\Delta_{01} > 0$, the alignment is Left-Right *regardless of the values of* Δ_{10} and Δ_{11}. Of course, the same is true when Δ_{10}, Δ_{11} are both positive. Likewise, whenever $\Delta_{00} < 0$ and $\Delta_{10} < 0$, this will mean that the two Down factions prefer each other best so that the Down-Up alignment will be stable, once again, regardless of the Up factions' preferences. The other cases of a unique stable alignment are exhausted by $\Delta_{01}, \Delta_{11} < 0$. Here, once again, the alignment will be Down-Up, in this case because the two Up factions prefer each other best.

There are two remaining cases which deserve special mention. They are: $\Delta_{00} > 0, \Delta_{01} < 0, \Delta_{10} < 0, \Delta_{11} > 0$ and $\Delta_{00} < 0, \Delta_{01} > 0, \Delta_{10} > 0, \Delta_{11} < 0$. In these cases, no couple exist such that its members prefer each other best. But this is only a *sufficient* condition for stability. We still need to check the *necessary* condition which is that of no pair possibly being able to "block" the alignment (to use the terminology of cooperative game theory), that is, to attain an improvement by its own. It is easy to verify that here *both* possible alignments meet this criterion. Under any of them, two factions will be matched with their preferred partners and the other two will rather prefer the other alignment. But these last two disgruntled factions can only opt out of the prevailing one if they can woo their other adjacent faction which happens to be matched with its best option. Therefore, once an alignment is established, it cannot be undone by bilateral agreements. This is the reason for which the solution is non-unique, which explains the labeling of these cases as cases of "indeterminacy" adopted in the paper.

A.2 Sketch of Proof for Proposition 2

An algebraic proof of the single-crossing property for the Δ functions would be very difficult and of little illustrative value. Here I will limit myself to providing a geometric proof for one of the possible cases in which the electoral equilibria may fall. The other cases work analogously.

To fix ideas, I will consider the case in which $x_L < x_R, y_L < y_R, x_C < x_D, y_C < y_D$. The most important step is to notice that, from Equations 2.13-2.16,2.21-2.24, the *loci* of the equilibria for each party platform are straight lines on the policy space with slope -1:

$$\begin{aligned} y_L &= \frac{1+\alpha_{01}}{\alpha_{00}+\alpha_{01}} - x_L \\ y_R &= \frac{2\alpha_{11}+\alpha_{10}-1}{\alpha_{11}+\alpha_{10}} - x_R \\ y_C &= \frac{1+\alpha_{10}}{\alpha_{00}+\alpha_{10}} - x_C \\ y_D &= \frac{\alpha_{01}+1}{\alpha_{01}+\alpha_{11}} - x_D \end{aligned}$$

It is also important to note that, as β varies from 0 to 1, the equilibrium platforms L and C move in the Southeast direction while the equilibrium platforms R and D do so in the Northwestern direction. Figure ?? depicts this situation.

Factions $(1,0),(1,1)$ are just the mirror image of factions $(0,0),(0,1)$ so let's consider the latter ones. The single-crossing property is very easy to establish for Δ_{01}: The diagram shows that $U_{01}(x_L, y_L)$ is a monotonically decreasing function of β while $U_{01}(x_C, y_C)$ is monotonically increasing in β. Clearly, this implies that the difference between both functions (i.e. Δ_{01}) satisfies the single-crossing property.

For faction $(0,0)$ some additional considerations are needed. The utility this faction derives from each alternative platform as β increases does not change monotonically. In fact, to the right of the 45° the utility of a platform increases (as it moves toward the Southwest) while to the left it decreases.

There is one straightforward case: $\alpha_{01} = \alpha_{10}$. Actually, this is the only case for which an easy algebraic conclusion can be derived. If we write Δ_{00} as the difference between indirect utility functions we obtain the following:

$$\begin{aligned}
\Delta_{00} &= U_{00}(x_L(\alpha_{00},\alpha_{01},\beta), y_L(\alpha_{00},\alpha_{01},\beta)) - U_{00}(x_C(\alpha_{00},\alpha_{10},\beta), y_C(\alpha_{00},\alpha_{10},\beta)) \\
&= \left(\frac{2}{(\alpha_{10}+\alpha_{00})^2} - \frac{2}{(\alpha_{01}+\alpha_{00})^2}\right)\beta^2 + \left(\frac{2(1-\alpha_{10})}{(\alpha_{00}+\alpha_{10})^2} - \frac{2(1+\alpha_{01})}{(\alpha_{00}+\alpha_{01})^2}\right)\beta + \\
&\quad \frac{1+\alpha_{10}^2}{(\alpha_{00}+\alpha_{10})^2} - \frac{(1+\alpha_{01})^2}{(\alpha_{00}+\alpha_{01})^2}
\end{aligned}$$

Notice, that when $\alpha_{01} = \alpha_{10}$, this expression becomes linear in β so that *a fortiori* the single-crossing property holds. More exactly, the crossing occurs at $\beta = 0.5$.

Geometrically, in this case the two equilibrium loci (that of L and that of C) belong to the same straight line. Moreover, the *magnitude* of the displacement between $\beta = 0$ and $\beta = 1$ is the same in both cases. (See, for example Figure ??.) It should also be noticed that the peaks of both utility profiles are equidistant from $\beta = 0.5$: while platform C reaches the 45° ray at some value $\tilde{\beta_C} < 0.5$, platform L does so at $\tilde{\beta_L} > 0.5$ and $\tilde{\beta_L} - 1/2 = 1/2 - \tilde{\beta_C}$. As by-product, this fact, together with the quadratic structure of U_{00} implies that:

$$\begin{aligned}
U_{00}(x_L(\alpha_{00},\alpha_{01},0), y_C(\alpha_{00},\alpha_{01},0) &< U_{00}(x_L(\alpha_{00},\alpha_{01},1), y_L(\alpha_{00},\alpha_{01},1) \\
U_{00}(x_C(\alpha_{00},\alpha_{10},0), y_C(\alpha_{00},\alpha_{10},0) &> U_{00}(x_C(\alpha_{00},\alpha_{10},1), y_C(\alpha_{00},\alpha_{10},1)
\end{aligned}$$

In fact, this last implication will hold for any set of values $\alpha_{00}, \alpha_{01}, \alpha_{10}$. Now, let's see what happens as one of the parameters (say, α_{10}) increases. This will produce a *left-ward shift* of the whole platform locus for C. In its turn, this will mean that the utility profile $U_{00}(x_C, y_C)$ decreases monotonically as a function of β. But at the same time, the path followed by the platform shortens. Formally, if $\alpha'_{01} > \alpha_{01}$, let $(x_C(\beta), y_C(\beta)) = (x_C(\alpha_{00},\alpha_{01},\beta), y_C(\alpha_{00},\alpha_{01},\beta)), (x'_C(\beta), y'_C(\beta)) = (x_C(\alpha_{00},\alpha'_{01},\beta), y_C(\alpha_{00},\alpha'_{01},\beta))$. Then, denoting the Euclidean distance operator as $d(\cdot,\cdot)$:

$$d((x_C(0), y_C(0)), (x_C(1), y_C(1)) < d((x'_C(0), y'_C(0)), (x'_C(1), y'_C(1))$$

This also implies that $\tilde{\beta_C}$ decreases. Geometrically, this is due to the fact that the segment of the locus left of the 45° is shorter and will be swept sooner as β increases.

Making all this algebraically precise is rather simple because the utiliity profiles are quadratic in β. However, since their respective coefficients can make the notation bewildering, some simplification is called for. From now on:

$$\begin{aligned} U_{00}(x_C, y_C) \equiv f(\beta) &= a_1\beta^2 + b_1\beta + c_1 \\ U_{00}(x_L, y_L) \equiv g(\beta) &= a_2\beta^2 + b_2\beta + c_2 \end{aligned}$$

Concavity of the utility profiles implies that $a_1, a_2 < 0$. On the other hand, $\tilde{\beta_C} < 1/2\tilde{\beta_L}$ amounts to say that $b_1 <| a_1 |<| a_2 |< b_2$. Hence:

$$\frac{b_1 - b_2}{2(a_2 - a_1)} > 1/2 \tag{A.1}$$

The crossings of Δ_{00} occur at the roots of the polynomial $g(\beta) - f(\beta)$, easily characterized by the quadratic formula:

$$\frac{b_1 - b_2 \pm \sqrt{(b_2 - b_1)^2 - 4(a_2 - a_1)(c_2 - c_1)}}{2(a_2 - a_1)}$$

Now, in the case we are considering, $\alpha_{01} < \alpha_{10}$, this leads to $a_2 < a_1$. So, there are two possibilities to be considered: either $c_2 > c_1$ or $c_2 < c_1$. The first case together with inequality A.1 means that one of the roots (viz. the largest one) of the polynomial will be > 1 which is to say that there is at most one crossing in the relevant interval $[0, 1]$.

The second case, $c_2 < c_1$ implies that $g(0) < f(0)$. But, on the other hand, $b_2 > b_1$ and $a_2 < a_1$ lead to $b_2 - b_1 > c_2 - c_1 + a_2 - a_1$ or, alternatively, $a_2 + b_2 + c_2 > a_1 + b_1 + c_1$. This last inequality is simply $g(1) > f(1)$. So, once again, in the interval $[0, 1], \Delta_{00}$ has at most one crossing.

Appendix B

Proofs of Results for Chapter 3

B.1 Proofs of Results

B.1.1 Proof of Lemma 2

Under strategic voting, the voters evaluate their candidates not by their declared local platform, but by the impact such platform will have on the final policy outcome. However, since there is electoral uncertainty, the voters need to evaluate their candidates' impact over all the possible legislatures. Therefore, we will need some notation for the probability of different legislatures being elected. Hence, $p(\mathbf{x})$ is the probability that legislature $\mathbf{x} = (x_1, \dots, x_N)$ is elected:

$$p(\mathbf{x}) = \prod_{A=1}^{N} p_A(x_A), \quad p_A(x_A) = \begin{cases} \pi_A(x_l, x_r) & \text{if} \quad x_A = x_{lA} \\ 1 - \pi_A(x_l, x_r) & \text{if} \quad x_A = x_{rA} \end{cases}$$

Since $x^*(\mathbf{x}_{-A}, x_A)$ is monotonic non-decreasing in x_A, then for all possible legislatures, $x^*(\mathbf{x}_{-A}, x_{lA}) \leq x^*(\mathbf{x}_{-A}, x_{rA})$. (For some legislatures, $x^*(\mathbf{x}_{-A}, x_{lA}) = x^*(\mathbf{x}_{-A}, x_{rA})$.)

Suppose, for the sake of clarity, that there are only two possible legislatures $\mathbf{x}'_{-A}, \mathbf{x}''_{-A}$ such that $x^*(\mathbf{x}'_{-A}, x_{lA}) < x^*(\mathbf{x}'_{-A}, x_{rA})$ and $x^*(\mathbf{x}''_{-A}, x_{lA}) < x^*(\mathbf{x}''_{-A}, x_{rA})$. (The extension to the general case poses no major difficulty.) For each of them, there is an indifferent voter

i', i'' respectively:

$$i' = \frac{x^*(\mathbf{x}'_{-A}, x_{lA}) + x^*(\mathbf{x}'_{-A}, x_{rA})}{2},$$

$$i'' = \frac{x^*(\mathbf{x}''_{-A}, x_{lA}) + x^*(\mathbf{x}''_{-A}, x_{rA})}{2},$$

Without loss of generality, suppose that $i' < i''$. Hence, for any type $i < i', E(u_i(x_{lA})) > E(u_i(x_{rA}))$ since, regardless of the actual legislature, those types belong to the set of supporters of l. Likewise, for any $i > i'', E(u_i(x_{lA})) < E(u_i(x_{rA}))$. For all i such that $i' < i < i''$:

$$u_i(\mathbf{x}'_{-A}, x_{lA}) < u_i(\mathbf{x}'_{-A}, x_{rA}),$$

$$u_i(\mathbf{x}''_{-A}, x_{lA}) > u_i(\mathbf{x}''_{-A}, x_{rA}),$$

Therefore, for every such i there exists a unique probability $p(\mathbf{x}'_{-A})$ for which:

$$\begin{aligned} E(u_i(x_{lA})) &= p(\mathbf{x}'_{-A})u_i(\mathbf{x}'_{-A}, x_{lA}) + (1 - p(\mathbf{x}'_{-A})u_i(\mathbf{x}''_{-A}, x_{lA}) \\ &= p(\mathbf{x}'_{-A})u_i(\mathbf{x}'_{-A}, x_{rA}) + (1 - p(\mathbf{x}'_{-A})u_i(\mathbf{x}''_{-A}, x_{rA}) \\ &= E(u_i(x_{rA})) \end{aligned}$$

By continuity of u, as i increases between i' and i'', $u_i(\mathbf{x}'_{-A}, x_{rA}) - u_i(\mathbf{x}'_{-A}, x_{lA})$ increases while $u_i(\mathbf{x}''_{-A}, x_{lA}) - u_i(\mathbf{x}''_{-A}, x_{rA})$ decreases. Therefore, the value $p(\mathbf{x}'_{-A})$ that renders i indifferent increases. The fact that this value p is unique for each i and increasing in i implies that for each $p(\mathbf{x}'_{-A})$ there is also a *unique* type i that is indifferent between the two candidates. This proves the lemma's claim.

B.1.2 Proof of Theorem 1

Since the policy outcome is determined by the median point of the elected legislature, the strategies chosen by candidates in one district depend upon the local platforms chosen in all the other districts. With $2N$ candidates and 2^N possible legislatures, a direct calculation of the equilibria is close to impossible. Therefore, I shall start by ruling out possible strategies.

Step 1: Non-deterministic policy outcomes First, I will prove that in equilibrium the location of the median legislator is uncertain. This result is important in itself because it rules out full convergence of the candidates to the median of their districts, a commonly assumed pattern in models of constituency elections.

Lemma 7 *Let $\mathcal{P}_O$ be an open-rule polity. In equilibrium, there is no policy outcome $x \in \Re$ such that* $\Pr(m(\mathbf{x}) = x) = 1$.

Proof: In such an equilibrium, L would be indifferent between endorsing and not endorsing all its candidates that propose $x_l > x$. If they are not endorsed, their district's platform will become τ_R but this will not affect the median. But, instead of randomizing its endorsement strategies in those districts, party L can in fact choose $e_L = 0$. That way, those candidates will be forced to propose platforms $x_l < x$ which would, if elected, increase L's pay-off. A similar reasoning would hold for party R.

There is one special case that needs to be considered: $x_{lA} = x_{rA} = x$ for all districts. In that case, no L candidate proposes a platform greater than x so that the preceding argument does not hold. However, this set of strategies cannot be an equilibrium. Suppose, without loss of generality, that $x > \mu_M$ (if $x \leq \mu_M$, the following argument goes through for the R candidates). Here, if endorsed, all the candidates have a probability of victory of 1/2. But, the L candidates of districts $A < M$ can improve (weakly) their pay-off by choosing platforms $x_{lA} = x - \epsilon$, for an $\epsilon > 0$ small enough. In fact, this would mean that, if elected, these candidates would yield a lower median which, in its turn, implies that the voters in their districts will no longer be indifferent and, then in those districts the pivotal voter will be in the interval $[x-\epsilon, x]$. With this pivotal location, the probability of victory of these candidates will be in the interval $[i_{mA}^{-1}(x-\epsilon), i_{mA}^{-1}(x)]$ and all these values are $> 1/2$.

Step 2: Range of possible medians In this step I will constrain the range in which the possible median legislators can be located.

Lemma 8 *Let $\underline{m}, \overline{m}$ denote the lowest and highest possible location of the median legislator in a political equilibrium of $\mathcal{P}_O$. Then:*

$$x_{lA} < \overline{m}, x_{rA} > \underline{m} \ \ \forall A$$

Proof: If an L candidate chooses $x_{lA} > \overline{m}$, party L will be indifferent between fielding her and allowing her district to return to the legislature a representative with platform τ_R. Therefore, she will be denied endorsement. The same holds for an R candidate that chooses $x_{rA} < \underline{m}$.

An implication of this result is that $\underline{m}$ is the median of a legislature formed only by L candidates and, likewise, $\overline{m}$ is the median when only R candidates are elected.

Lemma 9 *The range of possible median legislators is such that:*

$$\mu_{M-1} < \underline{m} < \mu_M < \overline{m} < \mu_{M+1}$$

Proof: If $\underline{m} > \mu_M$, this means that the indifferent voter in districts $A < M$ is: $i_A^* > \mu_M$ and, since $\mu_A < \mu_M$, this means that $i_{mA}^{-1}(\mu_M) > 1/2$. However, if an L candidate from those districts chooses a platform $\underline{m} < x_{lA} < \overline{m}$, her rival can choose $x_{rA} = x_{lA}$ ensuring a probability of victory of $1/2$. Notice that in so doing, the R candidate will retain endorsement because if that district elects a legislator with platform τ_L (which is what would happen if party R denies endorsement) this will shift the $\underline{m}$ to the left. Therefore, all L candidates from these districts choose a platform $x_{lA} < \underline{m}$. But this implies that the median of a legislature formed entirely by L candidates cannot be $\underline{m}$. This contradicts the condition on $\underline{m}$ derived in the last result. A similar contradiction, applied to R candidates will prove that $\overline{m} > \mu_M$.

The other inequalities can be proven by similar arguments: $\underline{m} < \mu_{M-1}$ requires that all the L candidates of districts 1 to $M-1$ propose platforms $x_l < \mu_{M-1}$. But this would imply that the L candidate in district $M-1$ can shift to the right the location of her district's pivotal voter (and hence, increase her probability of victory) by choosing a platform $x_{lM-1} > \underline{m}$. By the same token, we obtain that $\overline{m} < \mu_{M+1}$.

Step 3: Minimal Differentiation Here I will prove that, as the candidates attempt to reduce differentiation, in order to maximize their probability of victory, this narrows the interval of possible median locations.

Lemma 10 *In district M, the candidates choose platforms $x_{lM} = x_{rM} = \mu_M$.*

Proof: The logic used to prove Lemma 9 also shows that the candidates in district M do *not* benefit from choosing platforms out of $[\underline{m}, \overline{m}]$. So, this is the only district such that both candidates platforms' are contained in said interval. The crucial implication of this is that both candidates can secure endorsement even if their platforms converge within this range. In fact, district M will return the median legislator in at least one possible legislature: the one formed by L candidates from districts 1 to $M-1$ and R candidates from districts $M+1$ to N. So, if a party denies endorsement in district M this will shift the median against its preferences. This means that the usual argument of Downsian convergence applies here and $x_{lM} = x_{rM} = \mu_M$.

Up to this point, we have determined that, in equilibrium, the following pattern holds:

1. All the L candidates from districts 1 to $M-1$ propose platforms $x_l \leq \mu_{M-1}$.

2. All the R candidates from districts $M+1$ to N propose platforms $x_r \geq \mu_{M+1}$.

3. $x_{lM} = x_{rM} = \mu_M$.

4. All the L candidates from districts $M+1$ to N and all the R candidates from districts 1 to $M-1$ propose platforms in the interval (μ_{M-1}, μ_{M+1}).

This means that the candidates in the interval $[\underline{m}, \overline{m}]$ are *minority* candidates and, therefore, they maximize their probability of victory by approaching their rivals' strategy as much as is compatible with preserving the endorsement. In the case of the L candidates, this is accomplished by choosing $x_l = \mu_M - \epsilon$ while the R candidates choose $x_r = \mu_M + \epsilon$.

B.1.3 Proof of Theorem 3

The only part of the theorem that has not been proven yet is the conclusion $x_L^* \neq x_R^*$. In fact it includes two claims: one about existence of a Nash equilibrium in the convention stage and one about the actual nature of such Nash equilibrium, in particular, about policy divergence. I will discuss both of them in that order.

Existence: As Roemer [22] has pointed out, the pay-off function of the parties in this game is in general not quasi-concave. That means that a direct appeal to Kakutani's fixed point theorem is not possible because the best-response correspondences may fail to be convex valued. However, it is possible to restore the properties required for the application of the theorem. The first thing to notice is that the pay-off function of, say, party L can be rewritten as:

$$E(u_L(x_L)) = (u_L(x_L) - u_L(x_R))\Pi(x_L, x_R) + u_L(x_R)$$

Therefore, for any given strategy x_R, we know that L will never choose x_L such that $u_L(x_L) < u_L(x_R)$. Were it to do so, the first term would become negative and therefore, the pay-off would be inferior to the one obtained by choosing $x_L = x_R$. Since the policy preferences of the parties are represented by concave functions, then, the set $\tilde{x_L}(x_R) = \{x_L \mid u_L(x_L) \geq u_L(x_R)\}$ is a convex set. More precisely, $\tilde{x_L}(x_R) = [2\tau_L - x_R, x_R]$ (or $[x_R, 2\tau_L - x_R]$ if $x_R < \tau_L$).

On the other hand, $\Pi(x_L, x_R)$ is monotonic in x_L over the sets $x_L < x_R$ and $x_L > x_R$. To see why, let's first write down $\Pi(x_L, x_R)$ as a function of a specific $\pi_A(x_L, x_R)$:

$$\begin{aligned}
\Pi(x_L, x_R) &= \Pr(\#\{B : u_B(x_L) > u_B(x_R)\} \geq \frac{N+1}{2}) \\
&= \Pr(\#\{B : u_B(x_L) > u_B(x_R), B \neq A\} \geq \frac{N+1}{2}) + \\
&\quad \pi_A(x_L, x_R) \Pr(\#\{B : u_B(x_L) > u_B(x_R), B \neq A\} = \frac{N-1}{2})
\end{aligned}$$

In words, $\Pi(x_L, x_R)$, as a function of $\pi_A(x_L, x_R)$ is equal to the probability of L obtaining a majority in all the districts, excluding A plus the probability of L obtaining a tie in all those districts *and* breaking that tie in its favor by winning the elections in A.

Therefore, it is clear that:

$$\frac{\partial \Pi}{\partial \pi_A} = \Pr(\#\{B : u_B(x_L) > u_B(x_R), B \neq A\} = \frac{N-1}{2}) > 0$$

On the other hand, for $x_L < x_R$, we have that:

$$\frac{\partial \pi_A(x_L, x_R)}{\partial x_L} = \frac{\partial i_{mA}^{-1}(\frac{x_L + x_R}{2})}{\partial x_L} > 0$$

where the inequality follows from the fact that i_{mA}^{-1} is the inverse of a monotonically increasing function.

Likewise, we can conclude that $\frac{\partial \pi_A(x_L, x_R)}{\partial x_L} < 0$ if $x_L > x_R$. If we put these two expressions together, we obtain the derivative of the probability of victory with respect to x_L:

$$\frac{\partial \Pi(x_L, x_R)}{\partial x_L} = \sum_{A=1}^{N} \frac{\partial \Pi(x_L, x_R)}{\partial \pi_A(x_L, x_R)} \frac{\partial \pi_A(x_L, x_R)}{\partial x_L}$$

This derivative will be positive if $x_L < x_R$ and negative $x_L > x_R$. Anyway, we know that in $\tilde{x_L}(x_R), x_L$ is never $> x_R$ *and* $< x_R$ at the same time, therefore, when constrained to $\tilde{x_L}(x_R)$, the function $\Pi(x_L, x_R)$ is monotonic in x_L which means that it is also *quasi-concave.*

From this we can conclude that the pay-off function $E(u_L(x_L))$, *when constrained to* $\tilde{x_L}(x_R)$, is quasi-concave. In fact, remember that $E(u_L(x_L)) = (u_L(x_L) - u_L(x_R))\Pi(x_L, x_R) + u_L(x_R)$ and that $(u_L(x_L) - u_L(x_R))$ is positive in this subset. Therefore:

$$\frac{\partial E(u_L(x_L))}{\partial x_L} = (u_L(x_L) - u_L(x_R))\frac{\partial \Pi(x_L, x_R)}{\partial x_L} + u_L'(x_L)\Pi(x_L, x_R)$$

since $\Pi(x_L, x_R)$ is a monotonic function, its derivative dictates the sign of the first term. On the other hand, the concavity of u_L ensures that u_L' is monotonically decreasing. So, it is the case that we can always partition $\tilde{x_L}(x_R)$ into two convex subsets: one where $E(u_L(x_L))$ is increasing and one where it is decreasing. This ensures the quasi-concavity of the pay-off function when constrained to $\tilde{x_L}(x_R)$.

It is also true that the correspondence $\tilde{x_L}(x_R)$ is upper hemi-continuous. This can be seen by writing down its formula:

$$\tilde{x_L}(x_R) = \begin{cases} [x_R, 2\tau_L - x_R] & \text{if} \quad x_R < \tau_L \\ x_R & \text{if} \quad x_R = \tau_L \\ [2\tau_L - x_R, x_R] & \text{if} \quad x_R > \tau_L \end{cases}$$

So, it is immediate to verify that for any sequences $x_{R,n}, x_{L,n}$, if $x_{L,n} \in \tilde{x_L}(x_{R,n})$ $\forall n$ and, $x_{R,n} \to x_R$, then $x_L \in \tilde{x_L}(x_R)$ which is the definition of upper hemi-continuity.

Essentially the same arguments can be applied to the set of strategies for R so that it is possible to obtain a correspondence $\tilde{x_R}(x_L)$ which is upper hemi-continuous and convex valued and also with a pay-off function $E(u(x_R))$ quasi-concave over $\tilde{x_R}(x_L)$. Therefore, all the conditions for Kakutani's theorem are fulfilled and it is possible to claim that a Nash equilibrium (x_L^*, x_R^*) exists.

Policy Divergence Now, suppose that $x_L^* = x_R^*$ so that $\Pi(x_L^*, x_R^*) = 1/2$. It can be proven that in this case there will exist a profitable unilateral deviation for at least one of the two parties so that this cannot be a Nash equilibrium.

In particular, under total policy convergence, L's pay-off is $u_L(x_L^*)$. Now, consider an alternative policy $x'_L = x_L^* - \epsilon$ for some $\epsilon > 0$ small enough. Then, $u_L(x_L^*) < u_L(x'_L)$. On the other hand:

$$E(u_L(x^*(x'_L, x_R^*))) = u_L(x'_L)\Pi(x'_L, x_R^*) + u_L(x_L^*)\Pi(x_L^*, x_R^*)$$

Thus, x'_L constitutes a profitable deviation if $\Pi(x'_L, x_R^*) > 0$. This will be true unless $i_{mA}^{-1}\left(\frac{x'_L + x_R^*}{2}\right) = 0 \;\; \forall A$. So, if this condition does not hold, L can deviate from x_L^*.

Finally, suppose that such condition actually holds. Then, by a similar argument, it is easy to prove that R can profitably deviate to $x'_R = x_R^* + \epsilon$ for some $\epsilon > 0$ arbitrarily small. In fact, $u_R(x'_R) > u_R(x_R^*)$ so that R's pay-off increases if $1 - \Pi(x_L^*, x'_R) > 0$. Since, by assumption, $i_{mA}^{-1}\left(\frac{x'_L + x_R^*}{2}\right) = 0 \;\; \forall A$, for any $\epsilon > 0$, the only possibility of $1 - \Pi(x_L^*, x'_R) = 0$ is if $i_{mA}^{-1}\left(\frac{x_L^* + x'_R}{2}\right) = 0 \;\; \forall A$ for any ϵ. But this would imply that the location of the median voter is a degenerate random variable in *all* the districts something that violates the assumption of electoral uncertainty. Therefore, R has a profitable deviation and (x_L^*, x_R^*), where $x_L^* = x_R^*$, is not a political Nash equilibrium.

B.1.4 Proof of Theorem 4

The proof will proceed in two major steps. First, I will prove that the multi-district case is, in a crucial sense, analogous to the single-district case. Thus, the arguments used by

Roemer [22] to prove the single-district case can be used here. In the second step I will show how such line of reasoning goes.

Step 1. Intradistrict homogeneity in the multi-district case. The main goal of this step is to prove the following lemma:

Lemma 11 *Let $\mathcal{P}_{C,n}$ be a sequence of closed-rule polities with increasing intradistrict homogeneity. Then, for all x_L, x_R for which $\frac{x_L+x_R}{2} \neq \mu_M$:*

$$\lim_{n\to\infty} \Pi_n(x_L, x_R) \in \{0, 1\}$$

Remark: In words, this lemma claims that as intradistrict homogeneity decreases, the sequence of polities converges to one without electoral uncertainty.

Proof: This follows simply from the fact that $\mathcal{P}_{C,n+1}$ is obtained by a m.p.r.r. of the distributions G_n of $\mathcal{P}_{C,n}$.

Without loss of generality, assume that $\mu_{M+q} > \frac{x_L+x_R}{2} > \mu_M$, for some integer $q \geq 1$. Then, $\pi_{1,n}(x_L, x_R) \geq \ldots \geq \pi_{M,n}(x_L, x_R) \geq \pi_{M+q,n}(x_L, x_R) \geq \ldots \geq \pi_{N,n}(x_L, x_R)$ for all n.

Since $G_{A,n+1}$ is a m.p.r.r. of $G_{A,n}$ for all A, then,

$$\lim_{n\to\infty} \pi_{A,n}(x_L, x_R) = \begin{cases} 1 & \text{if } A < M+q \\ 0 & \text{if } A \geq M+q \end{cases}$$

The probability of victory $\Pi_n(x_L, x_R)$ is obtained by a summation of terms of the form

$$\prod_{A \subseteq N} \pi_{A,n}(x_L, x_R) \prod_{B=N\setminus A} (1 - \pi_{B,n}(x_L, x_R))$$

where A contains all the possible combinations of a majority of districts ($\#A \geq \frac{N+1}{2}$). Therefore, the only term that does not converge to 0 is:

$$\lim_{n\to\infty} \Pi_n(x_L, x_R) = \lim_{n\to\infty} \prod_{A=1}^{M+q-1} \pi_{A,n}(x_L, x_R) \prod_{B=M+q}^{N} (1 - \pi_{B,n}(x_L, x_R)) \to 1$$

A similar argument can be made to prove that if $\frac{x_L+x_R}{2} < \mu_M$ then $\Pi_n(x_L, x_R) \to 0$.

Step 2a: Convergence of the pivotal location to μ_M. An important consequence of Step 1 is the following lemma:

Lemma 12 *Let $(x_L^*, x_R^*)_n$ be the equilibrium of polity $\mathcal{P}_{C,n}$ in the sequence. Then:*

$$\lim_{n\to\infty} \frac{x_{Ln}^* + x_{Rn}^*}{2} = \mu_M.$$

Proof: Without loss of generality, let's assume, to the contrary, that there exists a subsequence $(x_L^*, x_R^*)_{n_1}$ such that $\lim_{n_1\to\infty} \frac{x_{L,n_1}^*+x_{R,n_1}^*}{2} > \mu_M$. Let $\delta > 0$ be a constant and x'_{L,n_1} a sequence of platforms such that $\lim_{n_1\to\infty} x'_{L,n_1} = x'_L, x'_{L,n_1} < x^*_{L,n_1} \forall n_1$ and $\lim_{n_1\to\infty} \frac{x'_{L,n_1}+x^*_{R,n_1}}{2} > \mu_M$ with $u_L(x'_{L,n_1}) - u_L(x^*_{L,n_1}) = \delta > 0$. I claim that for all large $n_1, E(u_L(x^*(x'_{L,n_1}, x^*_{R,n_1}))) > E(u_L(x^*(x^*_{L,n_1}, x^*_{Rn_1})))$:

$$\begin{aligned}
&E(u_L(x^*(x'_{L,n_1}, x^*_{Rn_1}))) - E(u_L(x^*(x^*_{L,n_1}, x^*_{R,n_1}))) \\
&= (u_L(x'_{L,n_1}) - u_L(x^*_{R,n_1}))\Pi_{n_1}(x'_{L,n_1}, x^*_{R,n_1}) - (u_L(x^*_{L,n_1}) - u_L(x^*_{R,n_1}))\Pi_{n_1}(x^*_{L,n_1}, x^*_{R,n_1}) \\
&= \delta\Pi_{n_1}(x'_{L,n_1}, x^*_{R,n_1}) + (u_L(x^*_{L,n_1}) - u_L(x^*_{R,n_1}))(\Pi_{n_1}(x'_{L,n_1}, x^*_{R,n_1}) - \Pi_{n_1}(x^*_{L,n_1}, x^*_{R,n_1}))
\end{aligned}$$

Since the limit of the pivotal location is $> \mu_M$ for both sequences, we know from Lemma 11 that $\lim_{n_1\to\infty} \Pi_{n_1}(x^*_{L,n_1}, x^*_{R,n_1})) = \lim_{n_1\to\infty} \Pi_{n_1}(x'_{L,n_1}, x^*_{R,n_1}) = 1$. In turn, this implies that: $\lim_{n_1\to\infty} E(u_L(x^*(x'_{L,n_1}, x^*_{R,n_1}))) - E(u_L(x^*(x^*_{L,n_1}, x^*_{Rn_1}))) = \delta > 0$.

This establishes the claim and, therefore, the sequence x^*_{L,n_1}, x^*_{R,n_1} is not a sequence of equilibria.

Step 2b: Convergence of equilibrium platforms. Lemma 12 proves that along the sequence of equilibria, for n large enough, $x^*_{L,n} = \mu_M - a_n, x^*_{R,n} = \mu_M + b_n$, for $a_n, b_n > 0$ and that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$. Now we need to prove that the limit of both sequences is 0.

Assume, to the contrary, that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = c > 0$. Let there be a sequence of strategies $x'_{L,n}$ with $\lim_{n\to\infty} x'_{L,n} = x'_L$, such that $x^*_{R,n} > x'_{L,n} > x^*_{L,n}$ and $u_L(x'_{L,n}) - u_L(x^*_{R,n}) > 1/2(u_L(x^*_{L,n}) - u_L(x^*_{R,n})) > 0, \forall n$. Then, I will prove that for n large enough, $E(u_L(x^*(x'_{L,n}, x^*_{Rn}))) > E(u_L(x^*_{L,n}, x^*_{R,n}))), \forall n$.

We know that:

$$E(u_L(x^*(x'_{L,n}, x^*_{R,n}))) - E(u_L(x^*(x_{L,n}, x^*_{R,n}))) =$$
$$(u_L(x'_{L,n}) - u_L(x^*_{R,n}))\Pi_n(x'_{L,n}, x^*_{R,n}) - (u_L(x^*_{L,n}) - u_L(x^*_{R,n}))\Pi_n(x^*_{L,n}, x^*_{R,n})$$

On the other hand, $\lim_{n\to\infty} \frac{x'_{L,n}+x^*_{R,n}}{2} > \mu_M$ so that

$$1 = \lim_{n\to\infty} \Pi_n(x'_{L,n}, x^*_{R,n}) > \lim_{n\to\infty} \Pi_n(x^*_{L,n}, x^*_{R,n}) = 1/2$$

.

(The last inequality follows from the fact that μ_M is the median district.) Therefore:

$$\begin{aligned}
&\lim_{n\to\infty} E(u_L(x^*(x'_{L,n}, x^*_{R,n}))) - E(u_L(x^*(x^*_{L,n}, x^*_{R,n}))) \\
&\quad = (u_L(x'_L) - u_L(x^*_R)) - 1/2(u_L(x^*_L) - u_L(x^*_R)) \\
&\quad > 0
\end{aligned}$$

This inequality establishes that $x^*_{L,n}$ is not a sequence of best responses to $x^*_{R,n}$ and this contradiction proves the theorem.